

Neutrino Mass Spectrum, Dirac and Majorana Leptonic CP-Violation and Leptogenesis

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Compelling Evidences for ν -Oscillations

– ν_{atm} : **SK** UP-DOWN ASYMMETRY

θ_{23} -, L/E - dependences of μ -like events

Dominant $\nu_{\mu} \rightarrow \nu_{\tau}$ K2K, MINOS; CNRS (OPERA)

– ν_{\odot} : Homestake, Kamiokande, SAGE, GALLEX/GNO

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \rightarrow \nu_{\mu, \tau}$ BOREXINO; KamLAND..., LowNu

– LSND

Dominant $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$; MiniBOONE 11/04/07: **negative result**

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} \quad l = e, \mu, \tau.$$

The ν -Oscillation Data: 3- ν mixing

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} \quad l = e, \mu, \tau.$$

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21} , α_{31} - the two Majorana CP-violation phases.

S.M. Bilenky, J. Hosek, S.T.P.,1980
J. Schechter, J.W.F. Valle,1980;

- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.6 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.305$, $\cos 2\theta_{12} \gtrsim 0.26$ (2σ),
- $|\Delta m_{\text{atm}}^2| \equiv |\Delta m_{31}^2| \cong 2.4$ (2.5) $\times 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} < 0.040$ (0.056) 2σ (3σ).

A.Bandyopadhyay *et al.*, arXiv:0804.4857;

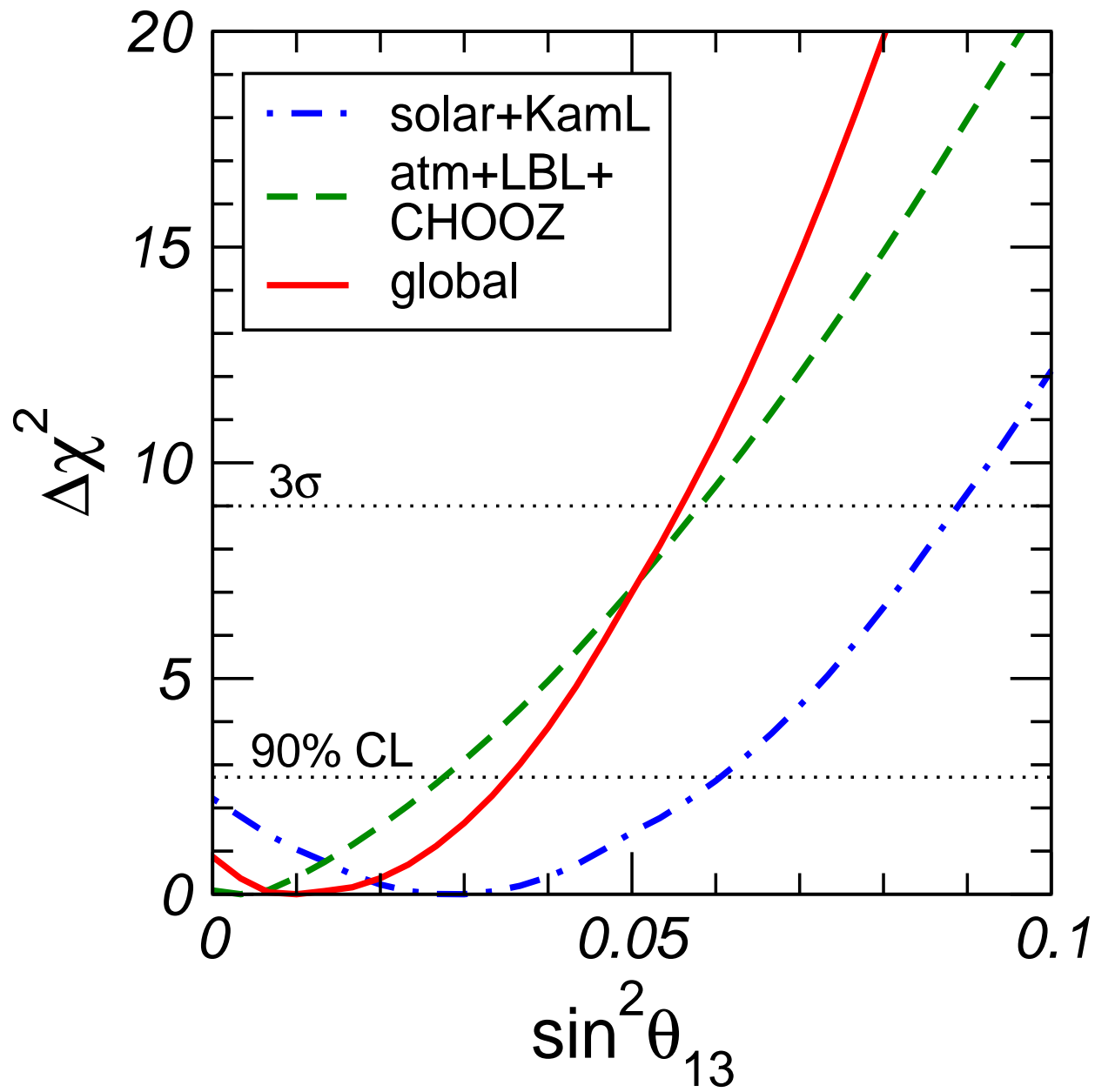
T. Schwetz *et al.*, arXiv:0808.2016

$$\sin^2 \theta_{13} = 0.016 \pm 0.010, \sin \theta_{13} = (0.077 - 0.161), 1\sigma$$

E. Lisi *et al.*, arXiv:0806.2649

Atmospheric ν data: $\cos \delta = -1$ favored over $\cos \delta = +1$

J. Escamilla *et al.*, arXiv:0805.2924



- $\text{sgn}(\Delta m_{\text{atm}}^2) = \text{sgn}(\Delta m_{31}^2)$ not determined

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0, \quad \text{normal mass ordering}$$

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - **NMO**, $m_3 < m_1 < m_2$ - **IMO**

$$m_1 \ll m_2 < m_3, \quad \text{NH,}$$

$$m_3 \ll m_1 < m_2, \quad \text{IH,}$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD; } m_j \gtrsim 0.10 \text{ eV.}$$

- Dirac phase δ : $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, l \neq l'$; $A_{\text{CP}}^{(l,l')} \propto J_{\text{CP}} \propto \sin \theta_{13} \sin \delta$

- Majorana phases α_{21}, α_{31} :

– $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P., 1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

– $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay depends on α_{21}, α_{31} ;

– $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;

– BAU, leptogenesis scenario: $\alpha_{21,31}$!

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle M(A,Z), \quad M(A,Z) - \text{NME},$$

$$|\langle m \rangle| \cong \left| \sqrt{\Delta m_{\odot}^2} \sin^2 \theta_{12} e^{i\alpha} + \sqrt{\Delta m_{31}^2} \sin^2 \theta_{13} e^{i\beta} \right|, \quad m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|\langle m \rangle| \cong \sqrt{m_3^2 + \Delta m_{13}^2} |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|\langle m \rangle| \cong m |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD)},$$

$$\theta_{12} \equiv \theta_{\odot}, \theta_{13} \text{-CHOOZ}; \quad \alpha \equiv \alpha_{21}, \beta + 2\delta \equiv \alpha_{31}.$$

CP-invariance: $\alpha = 0, \pm\pi, \beta_M = 0, \pm\pi;$

$$|\langle m \rangle| \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m_{13}^2} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m_{13}^2} \cong 0.055 \text{ eV, IH};$$

$$m \cos 2\theta_{12} \lesssim |\langle m \rangle| \lesssim m, \quad m \gtrsim 0.10 \text{ eV, QD}.$$

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .
- Determination of $\text{sgn}(\Delta m_{\text{atm}}^2)$, type of ν - mass spectrum

$$m_1 \ll m_2 \ll m_3, \quad \text{NH,}$$

$$m_3 \ll m_1 < m_2, \quad \text{IH,}$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD; } m_j \gtrsim 0.10 \text{ eV.}$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j -masses, or $\min(m_j)$.
- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?
- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2 \theta_{13}$.
- High precision determination of $\Delta m_{\odot}^2, \theta_{\odot}, \Delta m_{\text{atm}}^2, \theta_{\text{atm}}$.
- Searching for possible manifestations, other than ν_l -oscillations, of the non-conservation of $L_l, l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma, \tau \rightarrow \mu + \gamma$, etc. decays.

- Understanding at fundamental level the mechanism giving rise to the ν - masses and mixing and to the L_l -non-conservation. Includes understanding
 - the origin of the observed patterns of ν -mixing and ν -masses ;
 - the physical origin of CPV phases in U_{PMNS} ;
 - Are the observed patterns of ν -mixing and of $\Delta m_{21,31}^2$ related to the existence of a new symmetry?
 - Is there any relations between q -mixing and ν - mixing? Is $\theta_{12} + \theta_c = \pi/4$?
 - Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?
 - Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS} ?
- Progress in the theory of ν -mixing might lead to a better understanding of the origin of the BAU.
 - Can the Majorana and/or Dirac $CPVP$ in U_{PMNS} be the leptogenesis CPV parameters at the origin of BAU?

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ${}^3\text{H}$ β -decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \quad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN :} \quad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.7) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \quad \delta \cong 0.04 \text{ eV.}$$

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.

M. Gell-Mann, P. Ramond, R. Slansky, 1979;

T. Yanagida, 1979;

R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of ν -masses.
- Through **leptogenesis theory** links the ν -mass generation to the generation of baryon asymmetry of the Universe Y_B .

S. Fukugita, T. Yanagida, 1986.

- In SUSY GUT's with see-saw mechanism of ν -mass generation, the LFV decays

$$\mu \rightarrow e + \gamma, \quad \tau \rightarrow \mu + \gamma, \quad \tau \rightarrow e + \gamma, \quad \text{etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}$ -decay is allowed.

See-Saw: Dirac ν -mass m_D + Majorana mass M_R for N_R

The See-Saw Lagrangian

$$\mathcal{L}^{\text{lep}}(x) = \mathcal{L}_{\text{CC}}(x) + \mathcal{L}_Y(x) + \mathcal{L}_M^{\text{N}}(x),$$

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \bar{l}_L(x) \gamma_\alpha \nu_{lL}(x) W^{\alpha\dagger}(x) + h.c.,$$

$$\mathcal{L}_Y(x) = \lambda_{il} \bar{N}_{iR}(x) H^\dagger(x) \psi_{lL}(x) + Y_l H^c(x) \bar{l}_R(x) \psi_{lL}(x) + h.c.,$$

$$\mathcal{L}_M^{\text{N}}(x) = -\frac{1}{2} M_i \bar{N}_i(x) N_i(x).$$

ψ_{lL} - LH doublet, $\psi_{lL}^T = (\nu_{lL} \ l_L)$, l_R - RH singlet, H - Higgs doublet.

Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.

m_D generated by the Yukawa interaction:

$$-\mathcal{L}_Y^\nu = \lambda_{il} \bar{N}_{iR} H^\dagger(x) \psi_{lL}(x), \quad v = 174 \text{ GeV}, \quad v \lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2 \lambda^T M_R^{-1} \lambda = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger.$$

$Y_\nu \equiv \lambda = \sqrt{D_N} R \sqrt{D_\nu} (U_{\text{PMNS}})^\dagger / v_u$, all at M_R ; R -complex, $R^T R = 1$.

J.A. Casas and A. Ibarra, 2001

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

The CP-Invariance Constraints

Assume: $C(\bar{\nu}_j)^T = \nu_j$, $C(\bar{N}_k)^T = N_k$, $j, k = 1, 2, 3$.

The CP-symmetry transformation:

$$\begin{aligned} U_{\text{CP}} N_j(x) U_{\text{CP}}^\dagger &= \eta_j^{\text{NCP}} \gamma_0 N_j(x'), \quad \eta_j^{\text{NCP}} = i\rho_j^N = \pm i, \\ U_{\text{CP}} \nu_k(x) U_{\text{CP}}^\dagger &= \eta_k^{\nu\text{CP}} \gamma_0 \nu_k(x'), \quad \eta_k^{\nu\text{CP}} = i\rho_k^\nu = \pm i. \end{aligned}$$

CP-invariance:

$$\lambda_{jl}^* = \lambda_{jl} (\eta_j^{\text{NCP}})^* \eta^l \eta^{H*}, \quad j = 1, 2, 3, \quad l = e, \mu, \tau,$$

Convenient choice: $\eta^l = i$, $\eta^H = 1$ ($\eta^W = 1$):

$$\lambda_{jl}^* = \lambda_{jl} \rho_j^N, \quad \rho_j^N = \pm 1,$$

$$U_{lj}^* = U_{lj} \rho_j^\nu, \quad \rho_j^\nu = \pm 1,$$

$$R_{jk}^* = R_{jk} \rho_j^N \rho_k^\nu, \quad j, k = 1, 2, 3, \quad l = e, \mu, \tau,$$

λ_{jl} , U_{lj} , R_{jk} - either real or purely imaginary.

Relevant quantity:

$$P_{jkml} \equiv R_{jk} R_{jm} U_{lk}^* U_{lm}, \quad k \neq m,$$

$$\text{CP: } P_{jkml}^* = P_{jkml} (\rho_j^N)^2 (\rho_k^\nu)^2 (\rho_m^\nu)^2 = P_{jkml}, \quad \text{Im}(P_{jkml}) = 0.$$

$$P_{jkml} \equiv R_{jk} R_{jm} U_{lk}^* U_{lm}, \quad k \neq m,$$

$$CP: \quad P_{jkml}^* = P_{jkml} (\rho_j^N)^2 (\rho_k^\nu)^2 (\rho_m^\nu)^2 = P_{jkml}, \quad \text{Im}(P_{jkml}) = 0.$$

Consider NH N_j , NH ν_k : $P_{123\tau} = R_{12} R_{13} U_{\tau 2}^* U_{\tau 3}$

Suppose, CP-invariance holds at low E : $\delta = 0$, $\alpha_{21} = \pi$, $\alpha_{31} = 0$.

Thus, $U_{\tau 2}^* U_{\tau 3}$ - purely imaginary.

Then real $R_{12} R_{13}$ corresponds to CP-violation at "high" E .

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \quad (n_\gamma: \sim 6.3 \times 10^{-10})$$

$$Y_B \cong -10^{-2} \quad \varepsilon \kappa$$

W. Buchmüller, M. Plümacher, 1998;

W. Buchmüller, P. Di Bari, M. Plümacher, 2004

κ - efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε : CP -, L - violating asymmetry generated in out of equilibrium N_{Rj} -decays in the early Universe,

$$\varepsilon_1 = \frac{\Gamma(N_1 \rightarrow \Phi^- \ell^+) - \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}{\Gamma(N_1 \rightarrow \Phi^- \ell^+) + \Gamma(N_1 \rightarrow \Phi^+ \ell^-)}$$

M.A. Luty, 1992;

L. Covi, E. Roulet and F. Vissani, 1996;

M. Flanz *et al.*, 1996;

M. Plümacher, 1997;

A. Pilaftsis, 1997.

$\kappa = \kappa(\tilde{m})$, \tilde{m} - determines the rate of wash-out processes:

$\Phi^+ + \ell^- \rightarrow N_1$, $\ell^- + \Phi^+ \rightarrow \Phi^- + \ell^+$, etc.

W. Buchmüller, P. Di Bari and M. Plümacher, 2002;

G. F. Giudice *et al.*, 2004

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k} \right)}{\sum_j m_j |R_{1j}|^2}, \quad v = 174 \text{ GeV}$$

$$\widetilde{m}_l \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left| \sum_k R_{1k} m_k^{1/2} U_{lk}^* \right|^2, \quad l = e, \mu, \tau.$$

The “one-flavor” approximation - $\mathbf{Y}_{e,\mu,\tau}$ - “small”:

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l H^c(x) \bar{l}_R(x) \psi_{lL}$ - out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12}$ GeV

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{j,k} m_j^2 R_{1j}^2 \right)}{\sum_k m_k |R_{1k}|^2},$$

$$\widetilde{m}_1 = \sum_l \widetilde{m}_l = \sum_k m_k |R_{1k}|^2.$$

Two-Flavour Regime

At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not;

wash-out dynamics changes: τ_R^-, τ_L^+

$N_1 \rightarrow (\lambda_{1e} e_L^- + \lambda_{1\mu} \mu_L^- + \lambda_{1\tau} \tau_L^-) + \Phi^+$; $(\lambda_{1e} e_L^- + \lambda_{1\mu} \mu_L^- + \lambda_{1\tau} \tau_L^-) + \Phi^+ \rightarrow N_1$;

$\tau_L^- + \Phi^0 \rightarrow \tau_R^-$, $\tau_L^- + \tau_L^+ \rightarrow N_1 + \nu_L$, etc.

$\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently.

Three-Flavour Regime

At $M_1 \sim T \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\varepsilon_{1\tau}, \varepsilon_{1e}$ and $\varepsilon_{1\mu}$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: $L_\tau, \Delta L_\tau$ - distinguishable;

$L_e, L_\mu, \Delta L_e, \Delta L_\mu$ - individually not distinguishable;

$L_e + L_\mu, \Delta(L_e + L_\mu)$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1 (\sim T) \lesssim 10^{12}$ GeV,

$$\epsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k} \right)}{\sum_j m_j |R_{1j}|^2}$$

$$\widetilde{m}_l \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left| \sum_k R_{1k} m_k^{1/2} U_{lk}^* \right|^2, \quad l = e, \mu, \tau.$$

The baryon asymmetry is

$$Y_B \simeq -\frac{12}{37g_*} \left(\epsilon_2 \eta \left(\frac{417}{589} \widetilde{m}_2 \right) + \epsilon_\tau \eta \left(\frac{390}{589} \widetilde{m}_\tau \right) \right),$$

$$\eta(\widetilde{m}_l) \simeq \left(\left(\frac{\widetilde{m}_l}{8.25 \times 10^{-3} \text{ eV}} \right)^{-1} + \left(\frac{0.2 \times 10^{-3} \text{ eV}}{\widetilde{m}_l} \right)^{-1.16} \right)^{-1}.$$

$$Y_B = -(12/37) (Y_2 + Y_\tau),$$

$$Y_2 = Y_{e+\mu}, \quad \epsilon_2 = \epsilon_{1e} + \epsilon_{1\mu}, \quad \widetilde{m}_2 = \widetilde{m}_{1e} + \widetilde{m}_{1\mu}$$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R : $\varepsilon_{1l} \neq 0$, CPV from U

$$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0,$$

$$\begin{aligned} \varepsilon_{1\tau} &= -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{\tau j}^* U_{\tau k} R_{1j} R_{1k} \right)}{\sum_j m_j |R_{1j}|^2} \\ &= -\frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j) R_{1j} R_{1k} \text{Im} (U_{\tau j}^* U_{\tau k})}{\sum_j m_j |R_{1j}|^2}, R_{1j} R_{1k} = \pm |R_{1j} R_{1k}|, \\ &= \mp \frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j) |R_{1j} R_{1k}| \text{Re} (U_{\tau j}^* U_{\tau k})}{\sum_j m_j |R_{1j}|^2}, R_{1j} R_{1k} = \pm i |R_{1j} R_{1k}| \end{aligned}$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\text{Im} (U_{\tau j}^* U_{\tau k}) \neq 0$, $\text{Re} (U_{\tau j}^* U_{\tau k}) \neq 0$;

$$Y_B = -\frac{12}{37} \frac{\varepsilon_{1\tau}}{g_*} \left(\eta \left(\frac{390}{589} \widetilde{m}_\tau \right) - \eta \left(\frac{417}{589} \widetilde{m}_2 \right) \right)$$

$m_1 \ll m_2 \ll m_3, M_1 \ll M_{2,3}; R_{12}R_{13} - \text{real}; m_1 \cong 0, R_{11} \cong 0 (N_3 \text{ decoupling})$

$$\varepsilon_{1\tau} = - \frac{3M_1 \sqrt{\Delta m_{31}^2}}{16\pi v^2} \left(\frac{\Delta m_{\odot}^2}{\Delta m_{31}^2} \right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m_{\odot}^2}{\Delta m_{31}^2} \right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2} \\ \times \left(1 - \frac{\sqrt{\Delta m_{\odot}^2}}{\sqrt{\Delta m_{31}^2}} \right) \text{Im} (U_{\tau 2}^* U_{\tau 3})$$

$$\text{Im} (U_{\tau 2}^* U_{\tau 3}) = -c_{13} \left[c_{23}s_{23}c_{12} \sin \left(\frac{\alpha_{32}}{2} \right) - c_{23}^2 s_{12}s_{13} \sin \left(\delta - \frac{\alpha_{32}}{2} \right) \right]$$

$\alpha_{32} = \pi, \delta = 0: \text{Re}(U_{\tau 2}^* U_{\tau 3}) = 0, \text{CPV due to } R$

S. Pascoli, S.T.P., A. Riotto, 2006.

$$M_1 \ll M_2 \ll M_3, m_1 \ll m_2 \ll m_3 \text{ (NH)}$$

Dirac CP-violation

$$\alpha_{32} = 0 \text{ (} 2\pi \text{)}, \beta_{23} = \pi \text{ (} 0 \text{)}; \quad \beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \arg(R_{12}R_{13}).$$

$$|R_{12}|^2 \cong 0.85, |R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15 - \text{maximise } |\epsilon_\tau| \text{ and } |Y_B|:$$

$$|Y_B| \cong 2.8 \times 10^{-13} |\sin \delta| \left(\frac{s_{13}}{0.2} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right).$$

$$|Y_B| \gtrsim 8 \times 10^{-11}, \quad M_1 \lesssim 5 \times 10^{11} \text{ GeV imply}$$

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.11, \quad \sin \theta_{13} \gtrsim 0.11.$$

The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0 \text{ (} 2\pi \text{)}, \beta_{23} = 0 \text{ (} \pi \text{)}$:

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09, \quad \sin \theta_{13} \gtrsim 0.09; \quad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

$$M_1 \ll M_2 \ll M_3, m_1 \ll m_2 \ll m_3 \text{ (NH)}$$

Majorana CP-violation

$$\delta = 0, \text{ real } R_{12}, R_{13} (\beta_{23} = \pi (0));$$

$$\alpha_{32} \cong \pi/2, \quad |R_{12}|^2 \cong 0.85, \quad |R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15 - \text{maximise } |\epsilon_\tau| \text{ and } |Y_B|:$$

$$|Y_B| \cong 2 \times 10^{-12} \left(\frac{\sqrt{\Delta m_{31}^2}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right).$$

We get $|Y_B| \gtrsim 8 \times 10^{-11}$, for $M_1 \gtrsim 3.6 \times 10^{10} \text{ GeV}$, or $|\sin \alpha_{32}/2| \gtrsim 0.15$

$$M_1 \ll M_2 \ll M_3, m_3 \ll m_1 < m_2 \text{ (IH)}$$

$m_3 \cong 0, R_{13} \cong 0$ (N_3 decoupling): impossible to reproduce Y_B^{obs} for real $R_{11}R_{12}$;

$|Y_B|$ suppressed by the additional factor $\Delta m_{\odot}^2/|\Delta m_A^2| \cong 0.03$.

Purely imaginary $R_{11}R_{12}$: no (additional) suppression

Dirac CP-violation

$$\alpha_{21} = \pi; R_{11}R_{12} = i\kappa|R_{11}R_{12}|, \kappa = 1;$$

$|R_{11}| \cong 1.07, |R_{12}|^2 = |R_{11}|^2 - 1, |R_{12}| \cong 0.38$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

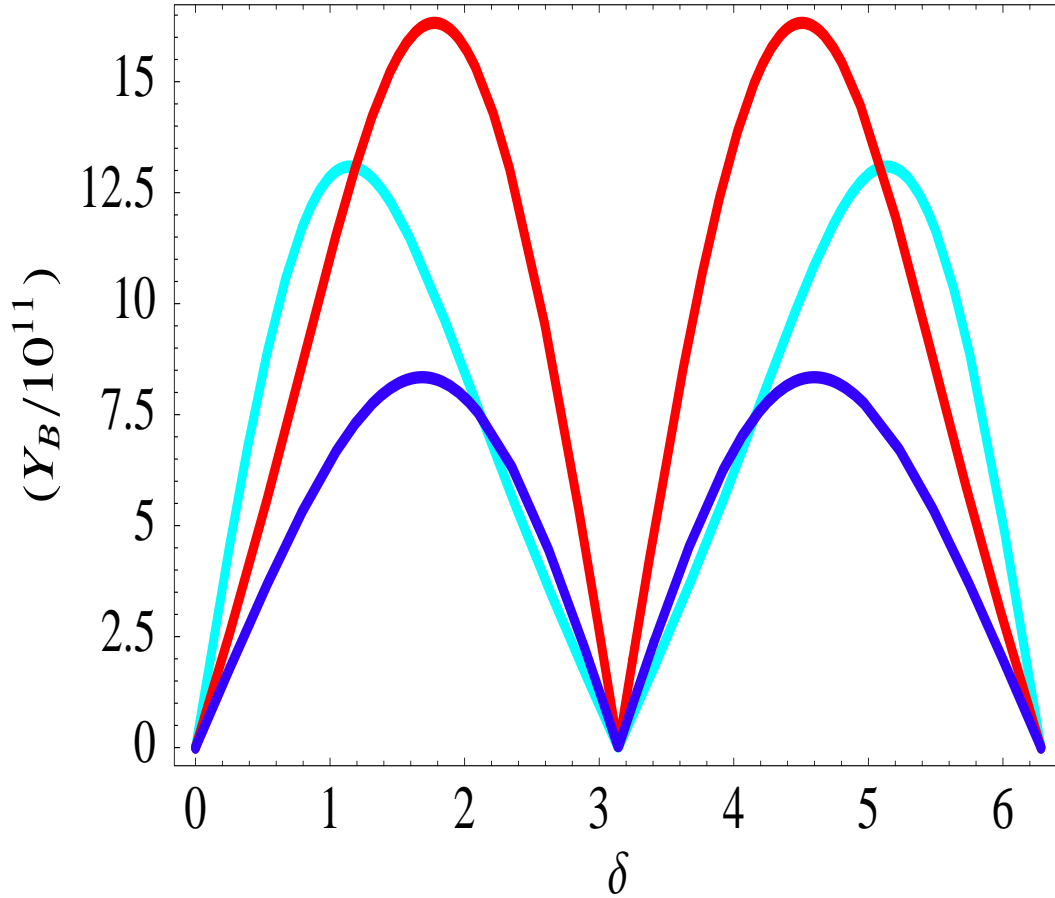
$$|Y_B| \cong 8.1 \times 10^{-12} |s_{13} \sin \delta| \left(\frac{M_1}{10^9 \text{ GeV}} \right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}, M_1 \lesssim 5 \times 10^{11} \text{ GeV}$ imply

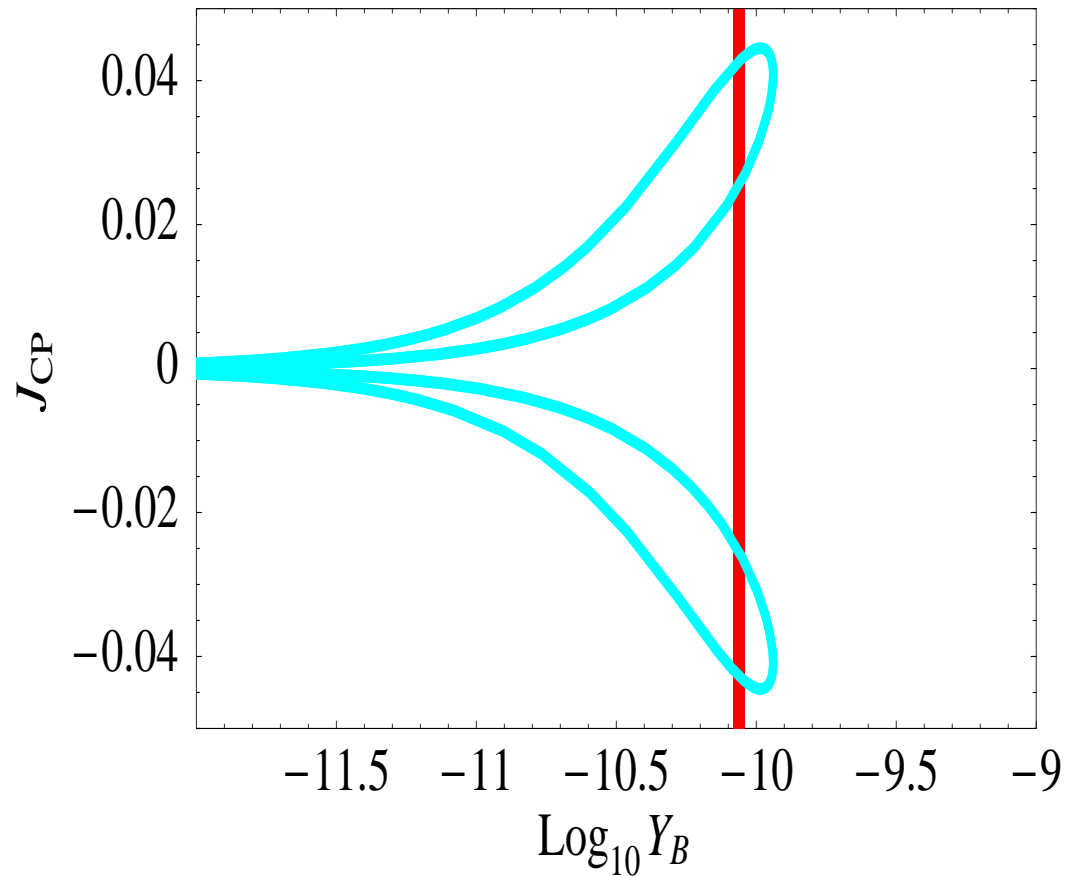
$$|\sin \theta_{13} \sin \delta| \gtrsim 0.02, \quad \sin \theta_{13} \gtrsim 0.02.$$

The lower limit corresponds to

$$|J_{CP}| \gtrsim 4.6 \times 10^{-3}$$

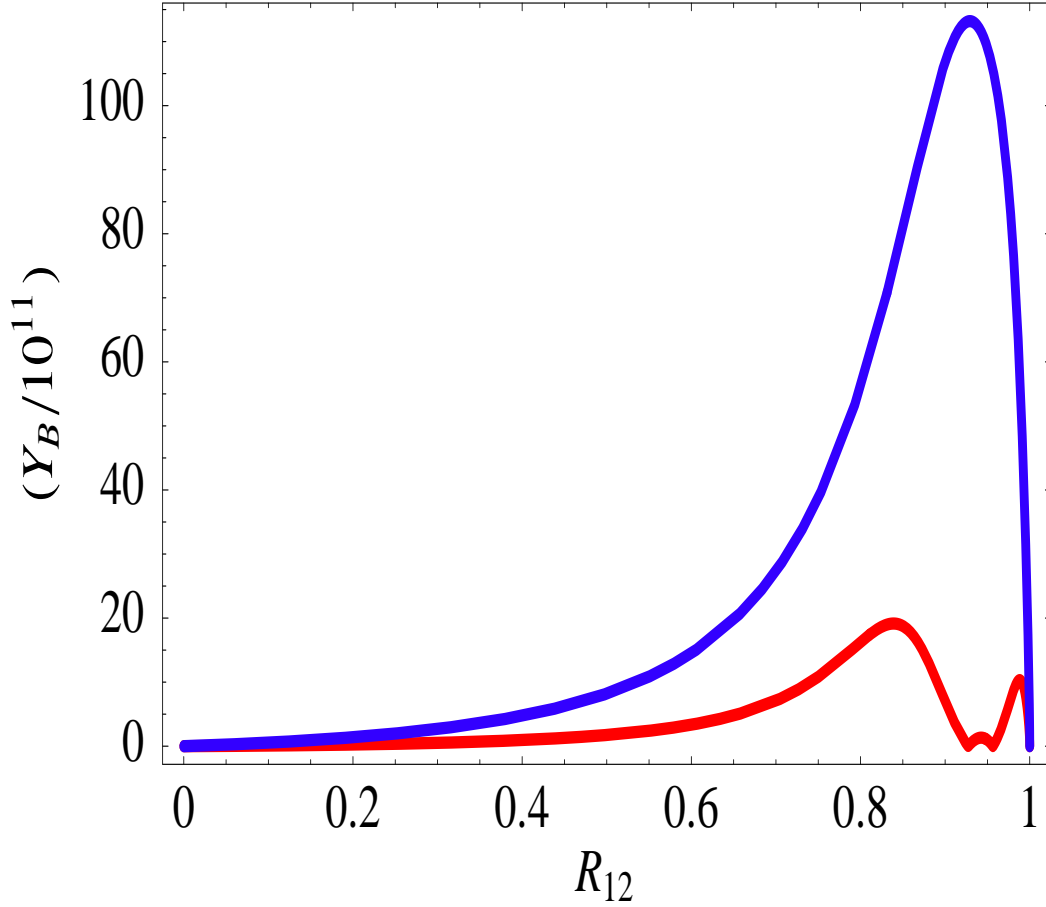


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Dirac CP-violation, $\alpha_{32} = 0$; 2π ;
 real R_{12} , R_{13} , $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, $\text{sign}(R_{12}R_{13}) = +1$;
 i) $\alpha_{32} = 0$ ($\kappa' = +1$), $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line);
 ii) $\alpha_{32} = 2\pi$ ($\kappa' = -1$), $s_{13} = 0.2$ (light blue line);
 $M_1 = 5 \times 10^{11}$ GeV.

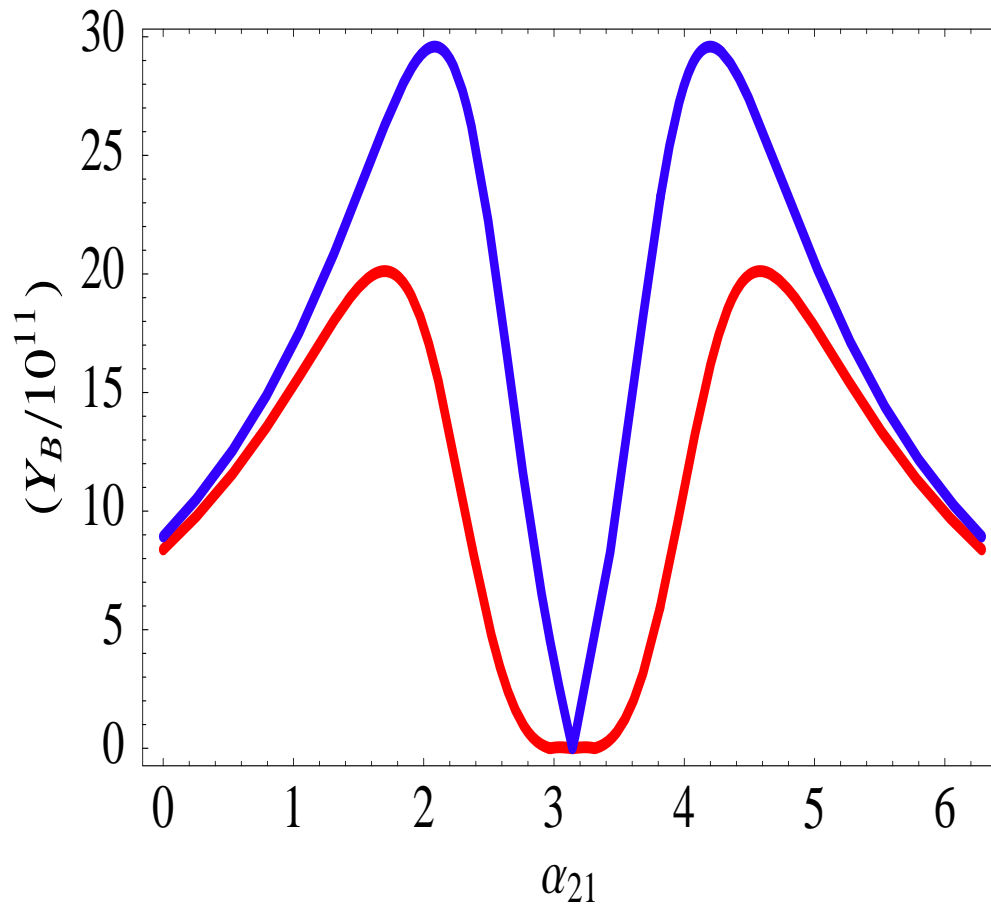


$M_1 \ll M_2 \ll M_3, m_1 \ll m_2 \ll m_3; M_1 = 5 \times 10^{11} \text{ GeV};$
 Dirac CP-violation, $\alpha_{32} = 0 \text{ (} 2\pi \text{)}$;
 $|R_{12}| = 0.86, |R_{13}| = 0.51, \text{sign}(R_{12}R_{13}) = +1 \text{ (-1)} \text{ (}\beta_{23} = 0 \text{ (}\pi\text{), } \kappa' = +1\text{)}$;
 The red region denotes the 2σ allowed range of Y_B .

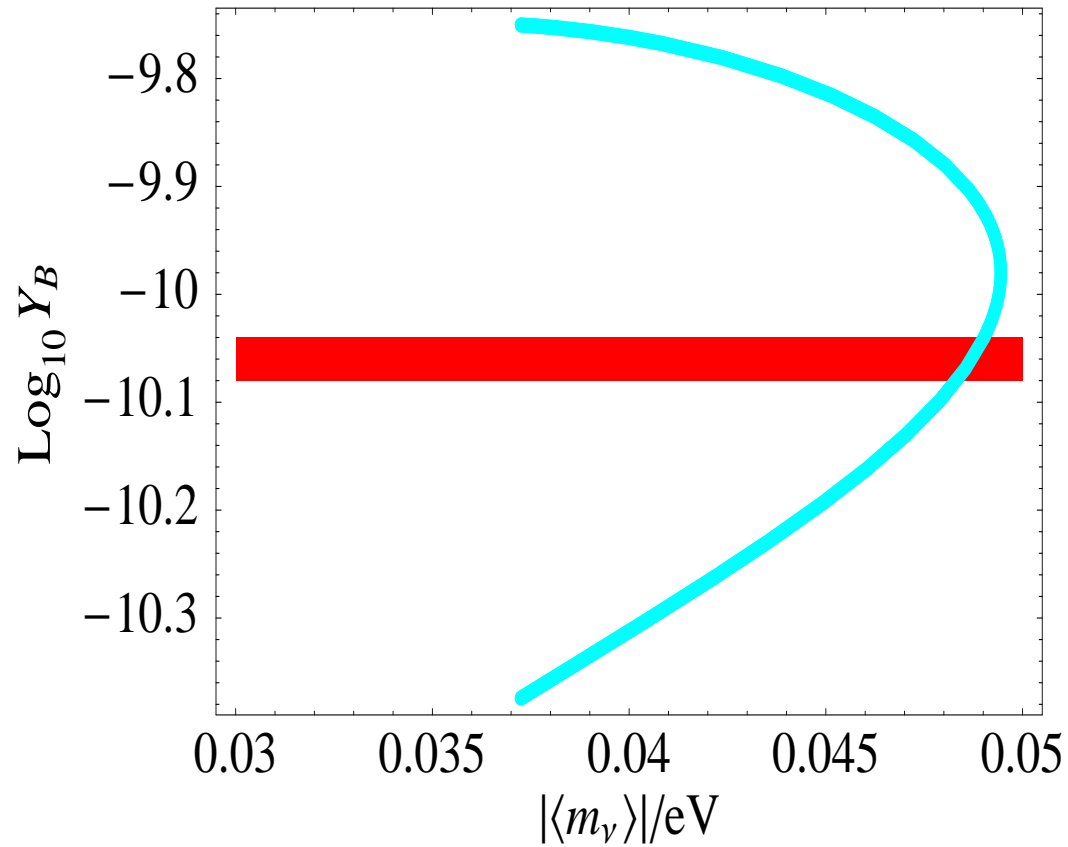
S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 \ll M_2 \ll M_3, m_1 \ll m_2 \ll m_3; M_1 = 5 \times 10^{11} \text{ GeV};$
 real $R_{12}, R_{13}, \text{sign}(R_{12}R_{13}) = +1, R_{12}^2 + R_{13}^2 = 1, s_{13} = 0.20;$
 a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ ($\kappa = +1$);
 b) Dirac CP-violation (red line), $\delta = \pi/2$ and $\alpha_{32} = 0$ ($\kappa' = +1$);
 $\Delta m_{\odot}^2, \sin^2 \theta_{12}, \Delta m_{31}^2, \sin^2 2\theta_{23}$ - fixed at their best fit values.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
 Majorana CP-violation, $\delta = 0$;
 purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
 $s_{13} = 0$ (blue line) and 0.2 (red line).



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
 Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;
 purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.
 The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3, m_3 \ll m_1 < m_2$ (IH)

Majorana or Dirac CP-violation

$m_3 \neq 0, R_{13} \neq 0, R_{11}(R_{12}) = 0$: possible to reproduce Y_B^{obs} for real $R_{12(11)} R_{13} \neq 0$

Requires $m_3 \cong (10^{-5} - 10^{-2})$ eV; non-trivial dependence of $|Y_B|$ on m_3

Majorana CPV, $\delta = 0$ (π): requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV

Dirac CPV, $\alpha_{32(31)} = 0$: typically requires $M_1 \gtrsim 10^{11}$ GeV

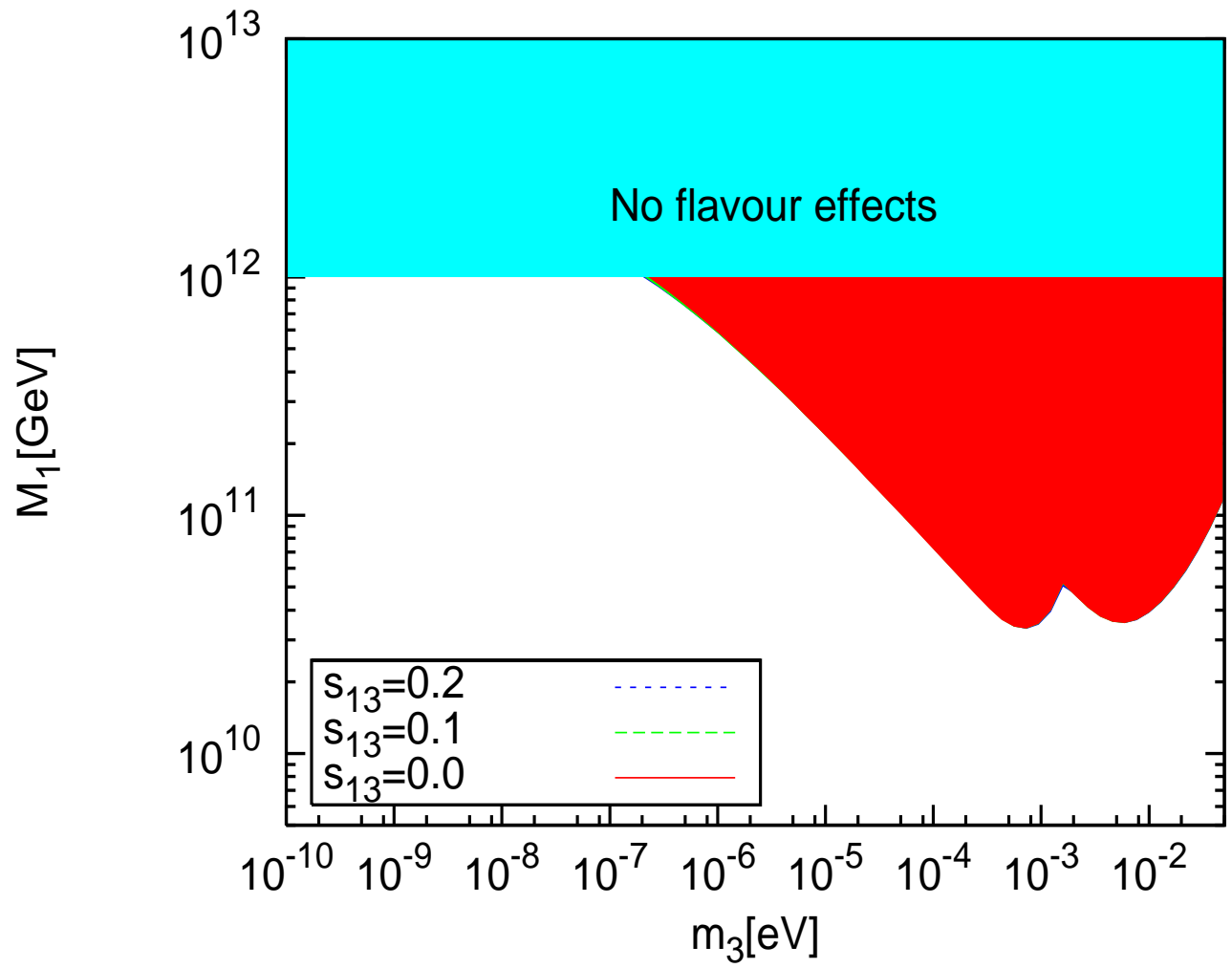
$|Y_B| \gtrsim 8 \times 10^{-11}, M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin \theta_{13} \sin \delta|, \sin \theta_{13} \gtrsim (0.04 - 0.09).$$

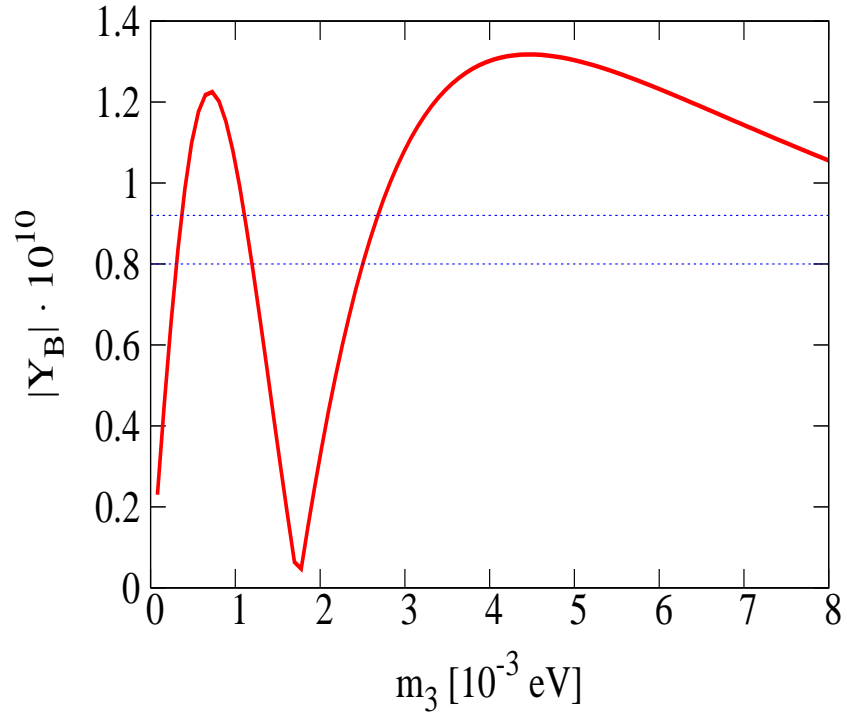
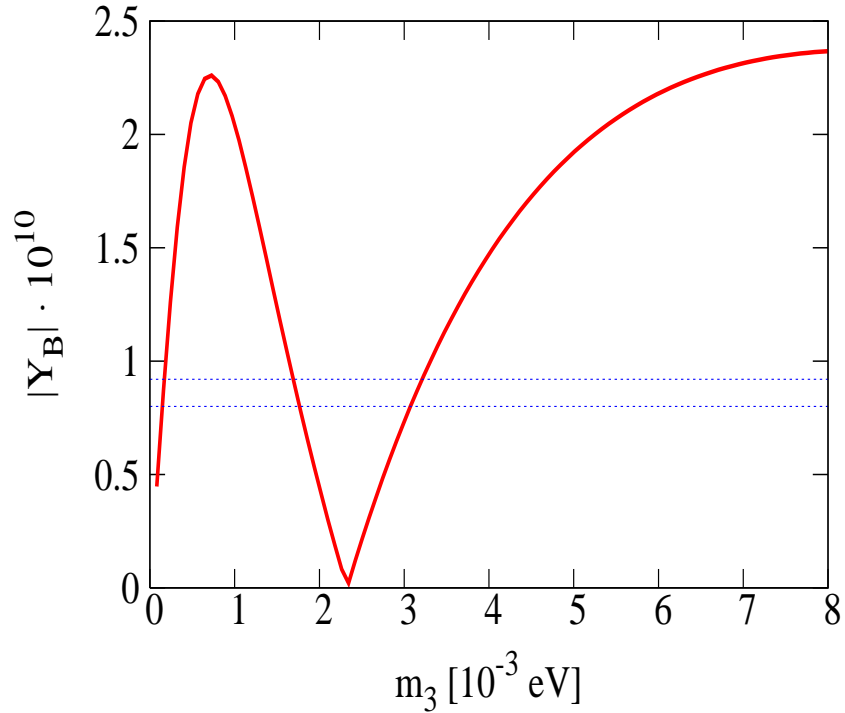
The lower limit corresponds to

$$|J_{CP}| \gtrsim (0.009 - 0.02)$$

NO (NH) spectrum, $m_1 < (\ll) m_2 < m_3$: similar dependence of $|Y_B|$ on m_1 if $R_{12} = 0, R_{11}R_{13} \neq 0$; non-trivial effects for $m_1 \cong (10^{-4} - 5 \times 10^{-2})$ eV.

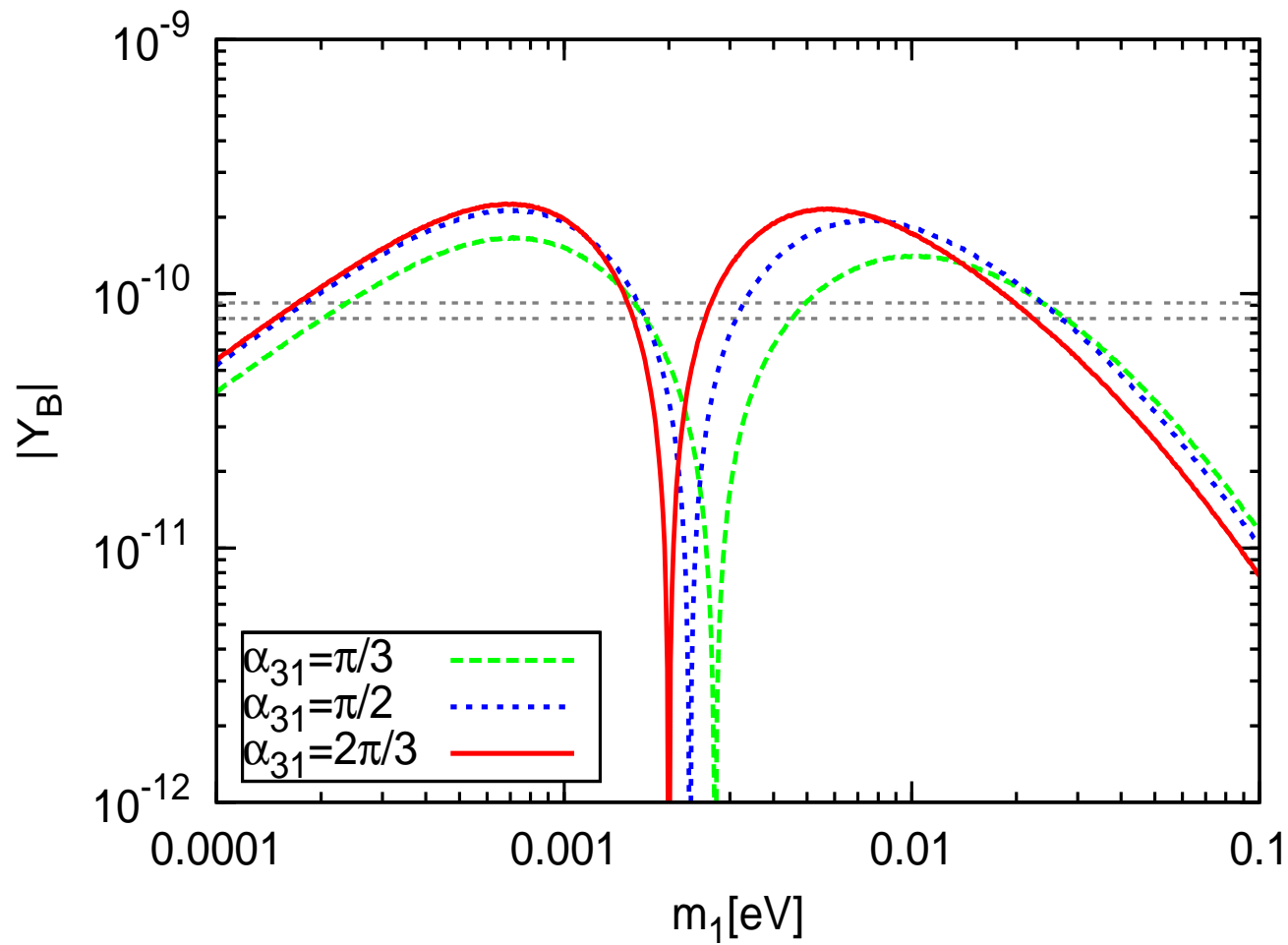


$m_3 < m_1 < m_2$, $M_1 \ll M_2 \ll M_3$, real R_{1j} ; $M_1 = (10^9 - 10^{12})$ GeV, $s_{13} = 0.2; 0.1; 0$;
 R_{1j} varied within $|R_{13}|^2 + |R_{12}|^2 + |R_{11}|^2 = 1$; $\alpha_{21}, \alpha_{31}, \delta$ varied in $[0, 2\pi]$;
 $\min(M_1)$ for given m_3 : $|Y_B| = 8.6 \times 10^{-11}$; absolute minima of M_1 :
 $m_3 \cong 5.5 \times 10^{-4}; 5.9 \times 10^{-3}$ eV, $\alpha_{32} \cong \pi/2$, $M_1 = 3.4 (3.5) \times 10^{10}$ GeV.



$m_3 \ll m_1 \ll m_2$ (IH), $R_{11} = 0$, real $R_{12}R_{13}$, Majorana CPV;
 $\alpha_{32} = \pi/2$, $s_{13} = 0$, $M_1 = 10^{11}$ GeV; $R_{12}^2/R_{13}^2 = m_3/m_2$: maximises $|\epsilon_\tau|$;
 i) $\text{sgn}(R_{12}R_{13}) = +1$; ii) $\text{sgn}(R_{12}R_{13}) = -1$.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



$m_1 < m_2 < m_3$ (NO(NH)), $R_{12} = 0$, real $R_{11}R_{13}$, Majorana CPV, $s_{13} = 0$;
 $\text{sgn}(R_{11}R_{13}) = -1$, $\sin^2 \theta_{23} = 0.50$, $M_1 = 1.5 \times 10^{11}$ GeV;
 $\alpha_{32} = 2\pi/3; \pi/2; \pi/3$ (red, blue, green lines).

Complex R : $\varepsilon_{1l} \neq 0$, CPV from U and R

$m_1 \ll m_2 < m_3$ (NH), $M_1 \ll M_{2,3}$; $m_1 \cong 0$, $R_{11} \cong 0$ (N_3 decoupling)

$$R_{12}^2 + R_{13}^2 = |R_{12}|^2 e^{i2\varphi_{12}} + |R_{13}|^2 e^{i2\varphi_{13}} = 1,$$

$$|R_{12}|^2 \sin 2\varphi_{12} + |R_{13}|^2 \sin 2\varphi_{13} = 0: \text{sgn}(\sin 2\varphi_{12}) = -\text{sgn}(\sin 2\varphi_{13}).$$

$$\cos 2\varphi_{12} = \frac{1+|R_{12}|^4-|R_{13}|^4}{2|R_{12}|^2}, \quad \sin 2\varphi_{12} = \pm \sqrt{1 - \cos^2 2\varphi_{12}},$$

$$\cos 2\varphi_{13} = \frac{1-|R_{12}|^4+|R_{13}|^4}{2|R_{13}|^2}, \quad \sin 2\varphi_{13} = \mp \sqrt{1 - \cos^2 2\varphi_{13}}.$$

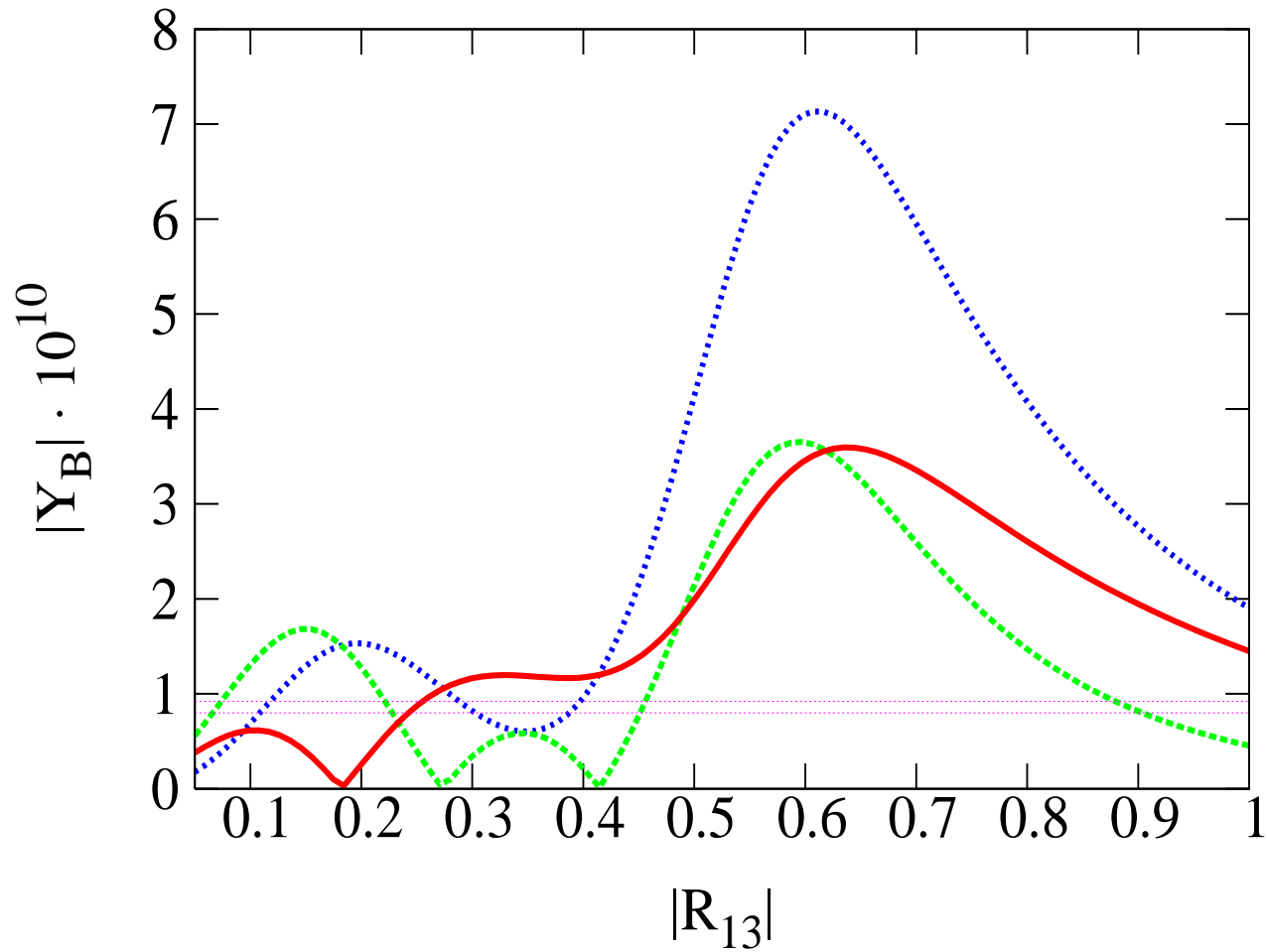
$m_3 \ll m_1 < m_2$ (IH), $M_1 \ll M_{2,3}$; $m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling)

$$R_{11}^2 + R_{12}^2 = |R_{11}|^2 e^{i2\varphi_{11}} + |R_{12}|^2 e^{i2\varphi_{12}} = 1,$$

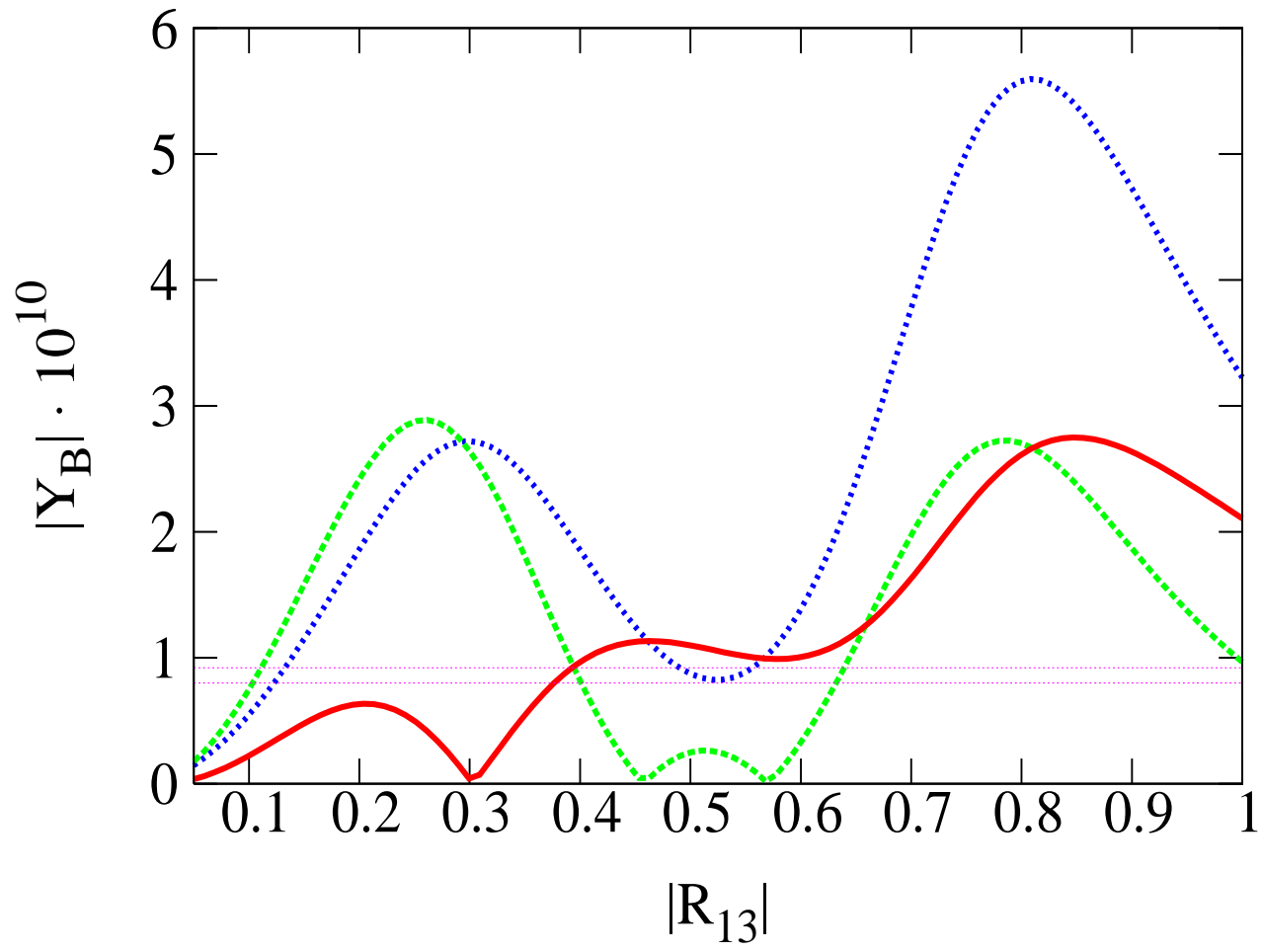
$$|R_{11}|^2 \sin 2\varphi_{11} + |R_{12}|^2 \sin 2\varphi_{12} = 0.$$

$|Y_B^0 A_{HE}| \propto |R_{11}|^2 \sin(2\varphi_{11}) (|U_{\tau 1}|^2 - |U_{\tau 2}|^2)$ - can be suppressed:

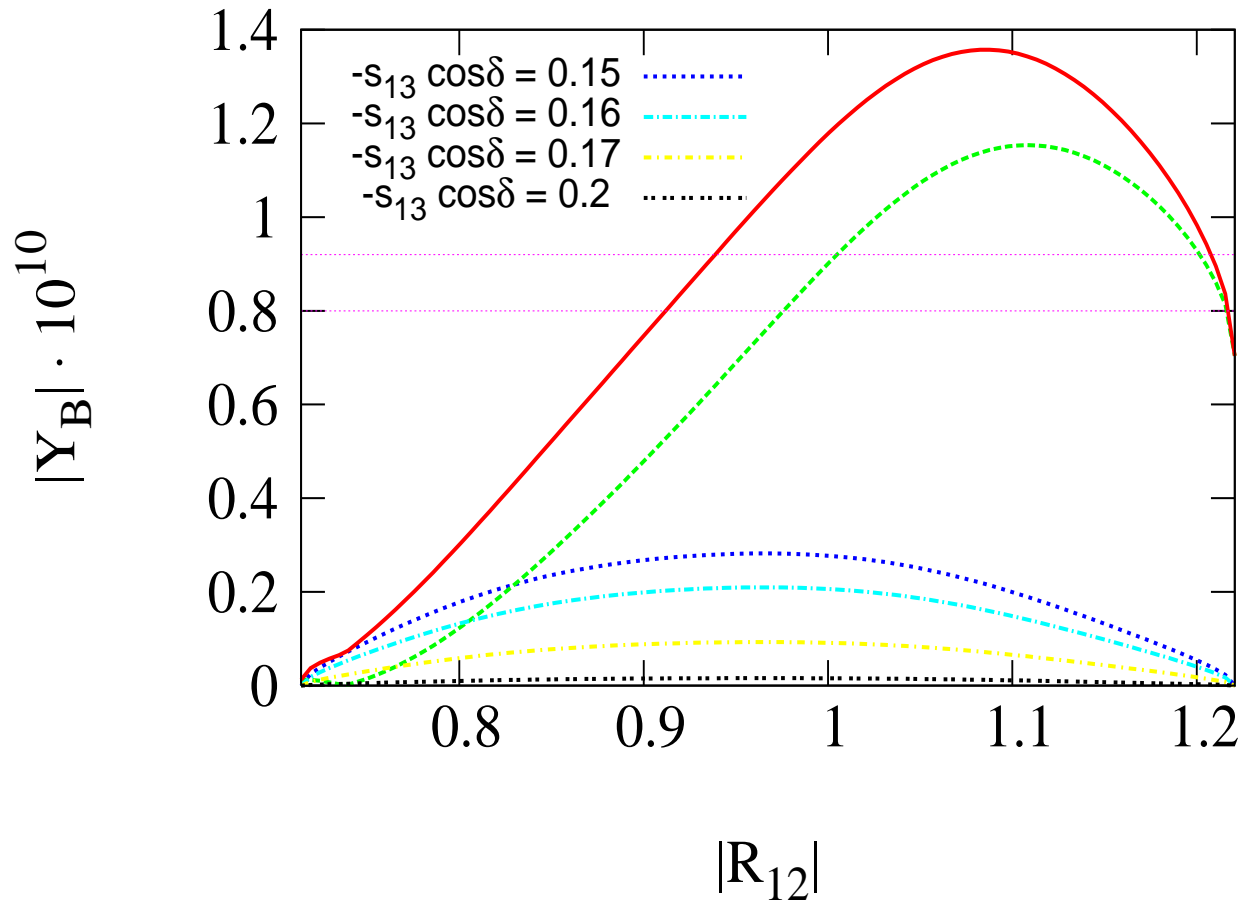
$$|U_{\tau 1}|^2 - |U_{\tau 2}|^2 \cong (s_{12}^2 - c_{12}^2)s_{23}^2 - 4s_{12}c_{12}s_{23}c_{23}s_{13} \cos \delta \cong -0.20 - 0.92 s_{13} \cos \delta.$$



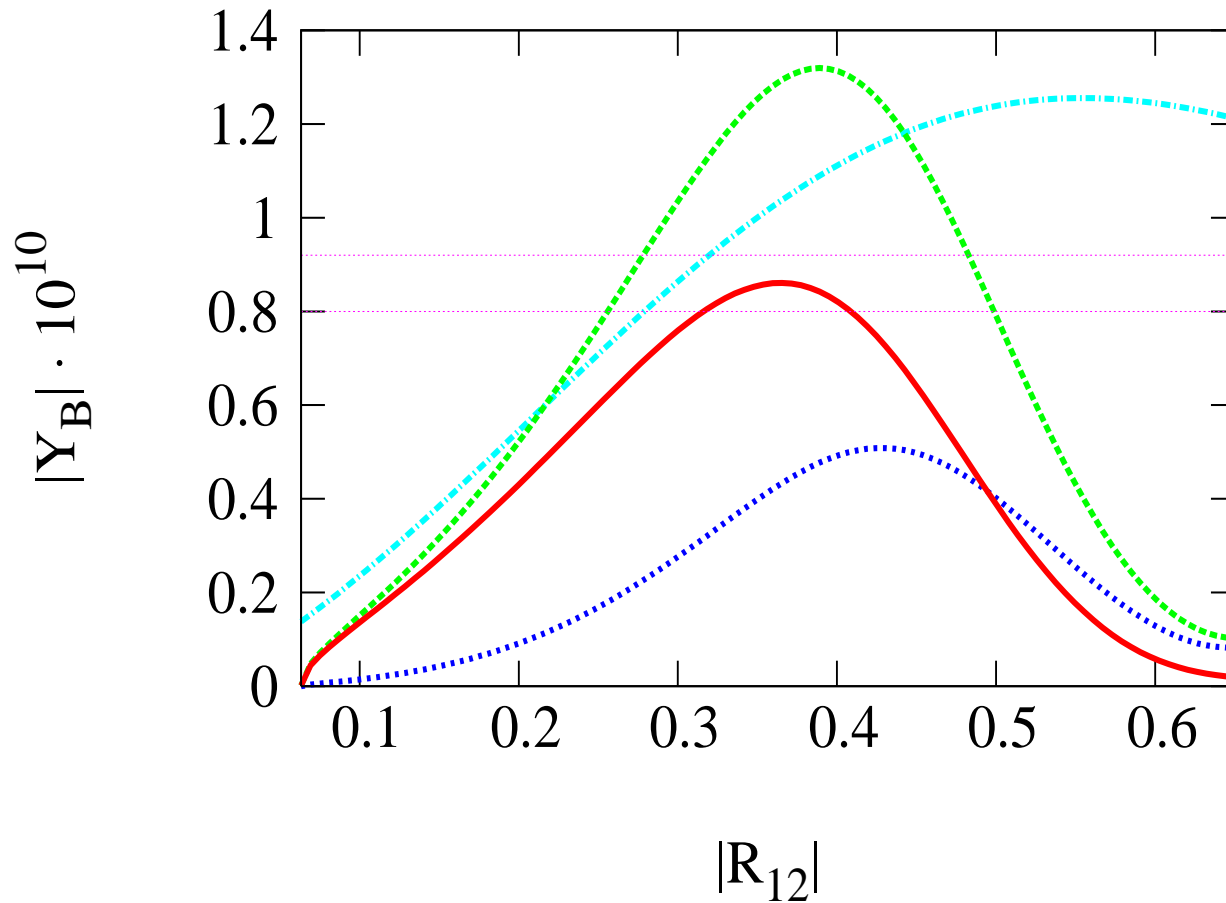
$m_1 < m_2 < m_3$ (NO(NH)), $R_{11} = 0$, CPV due to R and U ,
 $\alpha_{32} = \pi/2$, $s_{13} = 0$, $\sin^2 \theta_{23} = 0.50$, $M_1 = 10^{11}$ GeV;
 $|Y_B^0 A_{HE}|$ (R CPV, blue), $|Y_B^0 A_{MIX}|$ (U CPV, green), total $|Y_B|$ (red line)



$m_1 < m_2 < m_3$ (NO(NH)), $R_{11} = 0$, CPV due to R and U ,
 $\alpha_{32} = \pi/2$, $s_{13} = 0$, $\sin^2 \theta_{23} = 0.64$, $M_1 = 10^{11}$ GeV;
 $|Y_B^0 A_{HE}|$ (R CPV, blue), $|Y_B^0 A_{MIX}|$ (U CPV, green), total $|Y_B|$ (red line)



$m_3 \ll m_1 < m_2$ (IH), $R_{13} = 0$, Majorana and R -matrix CPV ,
 $\alpha_{21} = \pi/2$, $(-s_{13} \cos \delta) = 0.15$, $|R_{11}| = 1.2$, $M_1 = 10^{11}$ GeV;
 $|Y_B^0 A_{HE}|$ (R CPV, blue), $|Y_B^0 A_{MIX}|$ (U CPV, green), total $|Y_B|$ (red line).



$m_3 \ll m_1 < m_2$ (IH), $R_{13} = 0$, Majorana and R -matrix CPV ,
 $\alpha_{21} = \pi/2$, $s_{13} = 0$, $|R_{11}| \cong 1$, $M_1 = 10^{11}$ GeV;
 $|Y_B^0 A_{\text{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\text{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line) .
 Light-blue line: CP-conserving R , $R_{11}R_{12} \equiv ik|R_{11}R_{12}|$, $k = -1$ $|R_{11}|^2 - |R_{12}|^2 = 1$.

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν 's;

N_j, ν_k - Majorana particles

N_j : $M_1 \ll M_2 \ll M_3$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ in U_{PMNS} , no other sources of CPV (Majorana phases in U_{PMNS} equal to 0, etc.); requires $M_1 \gtrsim 10^{11}$ GeV.

$m_1 \ll m_2 \ll m_3$ (NH):

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09, \quad \sin \theta_{13} \gtrsim 0.09; \quad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

$m_3 \ll m_1 < m_2$ (IH):

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.02, \quad \sin \theta_{13} \gtrsim 0.02; \quad |J_{\text{CP}}| \gtrsim 4.6 \times 10^{-3}$$

B. CP-violation due to the Majorana phases in U_{PMNS} , no other sources of CPV (Dirac phase in U_{PMNS} equal to 0, etc.); requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV.

C. CP-violation due to both Dirac and Majorana phases in U_{PMNS} .

D. Y_B can depend non-trivially on $\min(m_j) \sim (10^{-5} - 10^{-2})$ eV.

S. Pascoli, S.T.P., A. Riotto, 2006 (A-C);
E. Molinaro, S.T.P., T. Shindou, Y. Takahashi, 2007 (D).

Conclusions

Determining the nature - Dirac or Majorana, of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses.

The see-saw mechanism provides a link between ν -mass generation and BAU. Majorana CPV phases in U_{PMNS} : $(\beta\beta)_{0\nu}$ -decay, Y_{B} .

Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Obtaining information on Dirac and Majorana CPV is a remarkably challenging problem.

Dirac and Majorana CPV may have the same source.

Low energy leptonic CPV can be directly related to the existence of BAU.

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

These results underline further the importance of the experiments aiming to measure the CHOOZ angle θ_{13} and of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.