Dirac–Born–Infeld and k-inflation: the CMB anisotropies from string theory

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Outline

L. Lorenz, J. Martin, CR: `arXiv:0807.3037`
L. Lorenz, J. Martin, CR: `arXiv:0807.2414`

Our universe as a 3-brane

- Current observations support inflationary cosmology
  - CMB power spectrum, flatness, gaussianity, adiabatic IC . . .
- Inflation can be induced by scalar fields in a flat enough $\ln[V(\varphi)]$
  - Slow-roll approach generically deals with any single field model
  - For minimal kinetic terms only
- String and M-theory live in more than four dimensions
  - Compact extra-dimensions
  - Contains branes + throats
  - A 3-brane in a throat
  - Its motion induces inflation
  - kinetic terms are modified

A summary of slow-roll inflation

- Friedmann–Lemaître background: $\mathrm{d}s^2 = a^2(\eta)(-\mathrm{d}\eta^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j)$
- Hubble flow functions [Schwarz 01]

$$N \equiv \ln a, \quad \epsilon_0 \equiv \frac{H_{\mathrm{in}}}{H}, \quad \epsilon_{n+1} \equiv \frac{\mathrm{d}\ln|\epsilon_n|}{\mathrm{d}N}$$

  - Equations of motion: accelerated expansion $\Leftrightarrow \epsilon_1 < 1$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}N} = -\frac{3-\epsilon_1}{3-\epsilon_1+\epsilon_2/2}\frac{\mathrm{d}\ln V}{\mathrm{d}\varphi}, \quad \epsilon_1 = \frac{1}{2}\left(\frac{\mathrm{d}\varphi}{\mathrm{d}N}\right)^2, \quad H^2 = \frac{V}{3-\epsilon_1}$$

  - Slow-roll trajectory: $\epsilon_i \ll 1 \Rightarrow N(\varphi) = -\int \frac{V}{V_{,\psi}}\mathrm{d}\psi$

- Inverting gives $\varphi(N)$ for any given potential V
  - Example: large field models $V \propto \varphi^p$

$$N = \frac{1}{2p}\left[\varphi_{\mathrm{ini}}^2 - \varphi^2(N)\right]$$

- Perturbed metric: scalar and tensor modes

$$\mathrm{d}s^2 = -a^2(1+2\Phi)\mathrm{d}\eta^2 + a^2\left[(1-2\Psi)\delta_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j$$

  - One scalar: comoving curvature (conserved on large scales):

$$\zeta = \Psi + \frac{\delta\phi}{\varphi_{,N}}, \qquad \zeta' = \frac{2}{\varphi_{,N}}\Delta\Psi$$

- Scalar and tensor modes evolution
  - Parametric oscillators

$$\left.\begin{aligned}\mu_{\mathrm{T}} &\equiv a h\\ \mu_{\mathrm{S}} &\equiv a\sqrt{2}\dot{\varphi}\zeta\end{aligned}\right\} \Rightarrow \mu_{\mathrm{TS}}'' + \left[k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}\right]\mu_{\mathrm{TS}} = 0$$

  - At leading order in slow-roll: Bessel functions

$$\frac{a''}{a} \simeq \frac{2+3\epsilon_{1*}}{\eta^2}, \qquad \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}} \simeq \frac{2+3\epsilon_{1*}+3\epsilon_{2*}/2}{\eta^2}$$

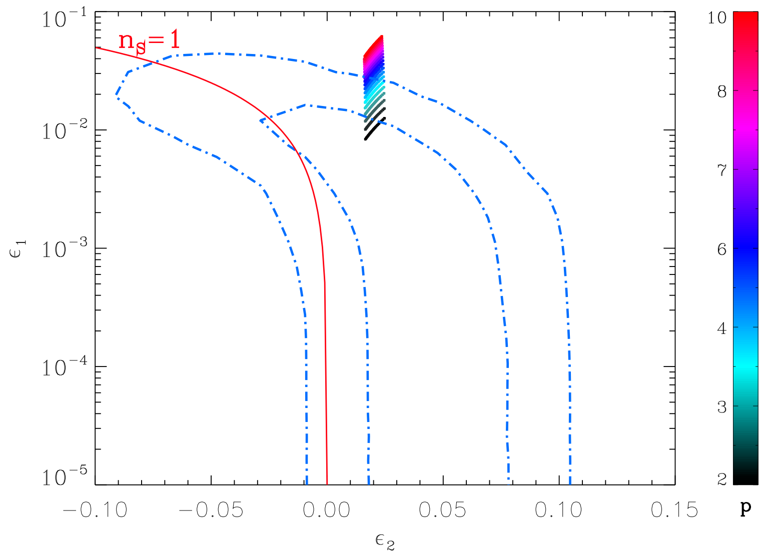
Seeds of CMB anisotropies

- Primordial power spectra: $\mathcal{P}_\zeta \equiv \frac{k^3}{2\pi^2}|\zeta|^2$ and $\mathcal{P}_h \equiv \frac{2k^3}{\pi^2}|h|^2$

$$\mathcal{P}_\zeta(k) = \frac{\kappa^2 H^2}{8\pi^2 \epsilon_{1*}}\left[1 - 2\left(C+1\right)\epsilon_{1*} - C\epsilon_{2*} - \left(2\epsilon_{1*} + \epsilon_{2*}\right)\ln\frac{k}{k_*}\right]$$

$$\mathcal{P}_h(k) = \frac{2\kappa^2 H^2}{\pi^2}\left[1 - 2\left(C+1\right)\epsilon_{1*} - 2\epsilon_{1*}\ln\frac{k}{k_*}\right]$$

- Comparison with the WMAP data



Kinetically driven inflation

- Generically, one may consider

$$S = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}\left[R + 2\kappa P\left(X,\varphi\right)\right], \qquad X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi.$$

- The resulting theory remains well defined for [Bruneton 07]
  - $\frac{\partial P}{\partial X} > 0$ and $2X\frac{\partial^2 P}{\partial X^2} + \frac{\partial P}{\partial X} > 0$
  - Example: DBI kinetic $\mathcal{L} = -T(\varphi)\sqrt{1 + g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi/T(\varphi)} + \ldots$
- New dynamics
  - Inflation driven by kinetic terms: k-inflation [Armendariz-Picon 99]
  - Speed of sound $c_\mathrm{s} \neq 1$: perturbations modified [Garriga 99]

$$c_\mathrm{s}^2 \equiv \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

K-inflationary perturbations

- Scalar mode equation in Fourier space
  - Comoving curvature perturbation $\mu_\mathrm{S}(k,\eta) \equiv \zeta\sqrt{2\epsilon_1}/c_\mathrm{S}$

$$\mu_\mathrm{S}'' + \left[c_\mathrm{S}^2 k^2 - \frac{(a\sqrt{\epsilon_1}/c_\mathrm{S})''}{a\sqrt{\epsilon_1}/c_\mathrm{S}}\right]\mu_\mathrm{S} = 0$$

  - Non-minimal terms: $c_\mathrm{S}(\eta)$ in the potential and frequency!
- Sound flow functions: $\delta_{i+1} = \dfrac{\mathrm{d}\ln|\delta_i|}{\mathrm{d}N}, \quad \delta_0 = \dfrac{c_{\mathrm{S\,ini}}}{c_\mathrm{S}}$
- The potential depends only on the ϵ_i and δ_i

$$\frac{(a\sqrt{\epsilon_1}/c_\mathrm{S})''}{(a\sqrt{\epsilon_1}/c_\mathrm{S})} = \mathcal{H}^2\Big[2 - \epsilon_1 + \frac{3}{2}\epsilon_2 + \frac{1}{4}\epsilon_2^2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{2}\epsilon_2\epsilon_3 + (3 - \epsilon_1 + \epsilon_2)\delta_1 + \delta_1^2 + \delta_1\delta_2\Big] \equiv \frac{\nu^2(\eta) - 1/4}{\eta^2}$$

Scalar mode evolution

- Slow-roll expansion around a time η_*
  - Constant potential ν_* at first order

$$\nu^2(\eta) = \frac{9}{4} + 3\epsilon_{1*} + \frac{3}{2}\epsilon_{2*} + 3\delta_{1*} + \mathcal{O}(\epsilon\delta) = \nu_*^2 + \mathcal{O}(\epsilon\delta)$$

  - Varying sound speed at first order

$$c_\mathrm{s}(\eta) = c_{\mathrm{s}*}\left(1 + \delta_{1*}\ln\frac{\eta}{\eta_*}\right) + \mathcal{O}(\epsilon\delta)\,.$$

- This is not a Bessel equation. Cannot be solved assuming c_s constant, even at first order! [Shandera 06, Peiris 07, Bean 07]

$$\mu_\mathrm{S}'' + \left[c_\mathrm{s}(\eta)^2 k^2 - \frac{\nu_*^2 - 1/4}{\eta^2}\right]\mu_\mathrm{S} = 0$$

- One can use the uniform approximation [Habib 04]

Primordial power spectra for k-inflation

- Scalar modes expanded around the time at which $-k_\diamond \eta_\diamond = 1/c_{\mathrm{s}\diamond}$

$$\mathcal{P}_\zeta \propto \frac{H_\diamond^2}{\pi m_{\mathrm{Pl}}^2 \epsilon_{1\diamond} c_{\mathrm{s}\diamond}} \Big[1 - 2(D+1)\epsilon_{1\diamond} - D\epsilon_{2\diamond} + (D+2)\delta_{1\diamond} - (2\epsilon_{1\diamond} + \epsilon_{2\diamond} - \delta_{1\diamond}) \ln \frac{k}{k_\diamond} \Big]$$

- Tensor power spectrum is unaffected
  - But the standard pivot is $-k_\diamond \eta_\triangleright = 1$

$$\mathcal{P}_h(k) \propto \frac{16 H_\triangleright^2}{\pi m_{\mathrm{Pl}}^2} \left[1 - 2(D+1)\epsilon_{1\triangleright} - 2\epsilon_{1\triangleright} \ln \frac{k}{k_\diamond} \right]$$

  - Switching to the sound horizon gives

$$\mathcal{P}_h(k) \propto \frac{16 H_\diamond^2}{\pi m_{\mathrm{Pl}}^2} \left[1 - 2\left(D + 1 + \ln \frac{1}{c_{\mathrm{s}\diamond}} \right) \epsilon_{1\diamond} - 2\epsilon_{1\diamond} \ln \frac{k}{k_\diamond} \right]$$

Corrected spectral index and running

- Spectral index up to second order: $n_\mathrm{s} - 1 = \left(\frac{\mathrm{d}\ln\mathcal{P}}{\mathrm{d}\ln k}\right)_{k=k_\diamond}$

$$n_\mathrm{s} - 1 = -2\epsilon_{1\diamond} - \epsilon_{2\diamond} + \delta_{1\diamond} - 2\epsilon_{1\diamond}^2 - (2D+3)\epsilon_{1\diamond}\epsilon_{2\diamond} + 3\epsilon_{1\diamond}\delta_{1\diamond} + \epsilon_{2\diamond}\delta_{1\diamond} - D\epsilon_{2\diamond}\epsilon_{3\diamond} - \delta_{1\diamond}^2 + (D+2)\delta_{1\diamond}\delta_{2\diamond}$$

- Running of the spectral index: $\alpha_\mathrm{s} = \left(\frac{\mathrm{d}^2\ln\mathcal{P}_\zeta}{\mathrm{d}\ln^2 k}\right)_{k=k_\diamond}$

$$\alpha_\mathrm{s} = -2\epsilon_{1\diamond}\epsilon_{2\diamond} - \epsilon_{2\diamond}\epsilon_{3\diamond} + \delta_{1\diamond}\delta_{2\diamond}$$

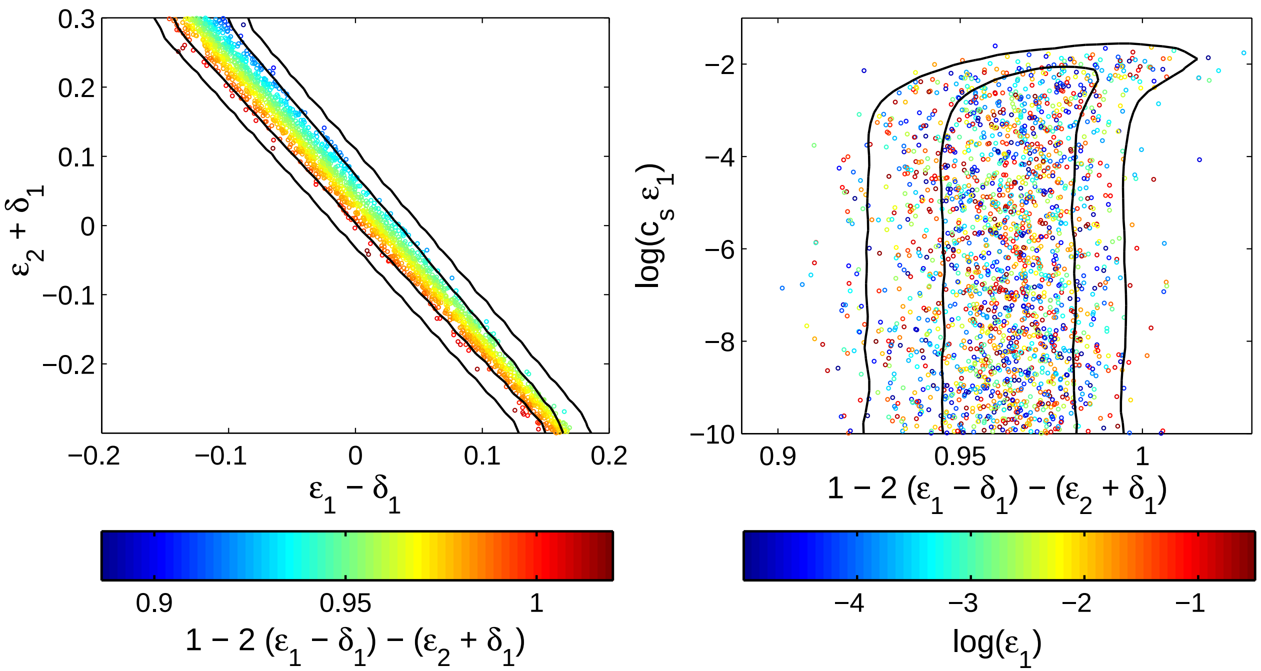
- Tensor to scalar ratio
  - Leading order

$$r = \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = 16 c_{\mathrm{s}\diamond}\epsilon_{1\diamond}$$

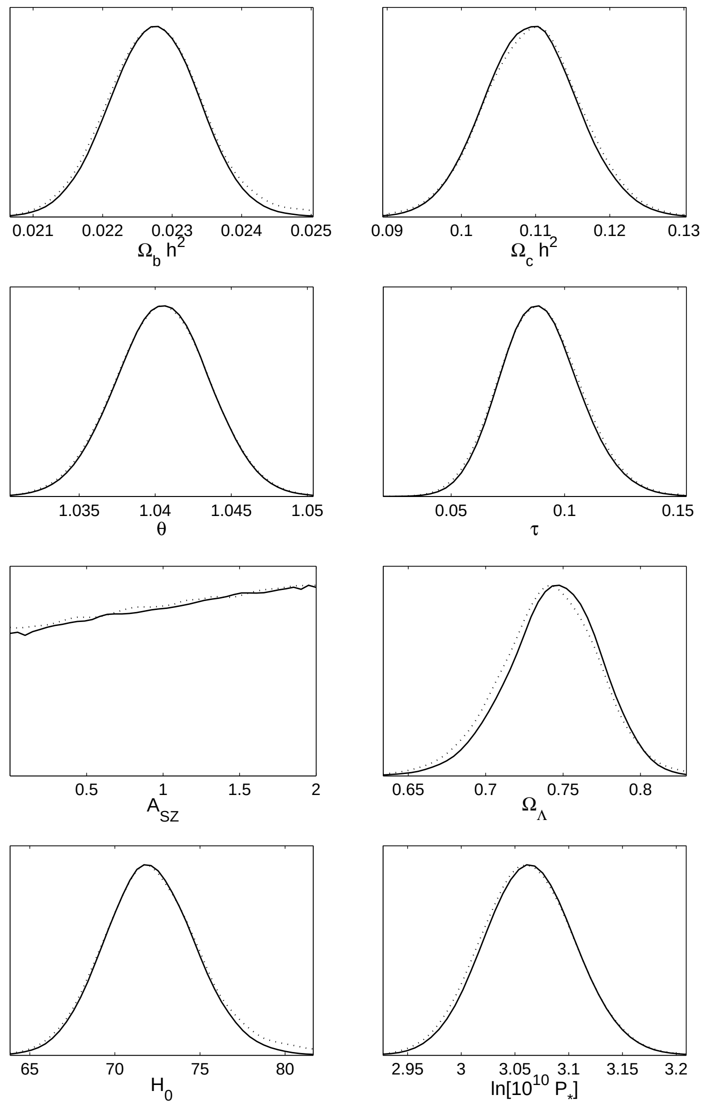
  - If $c_\mathrm{s} \ll 1$, one has to consider the $\ln(1/c_\mathrm{s})$ correction!

CMB data analysis in k-inflation

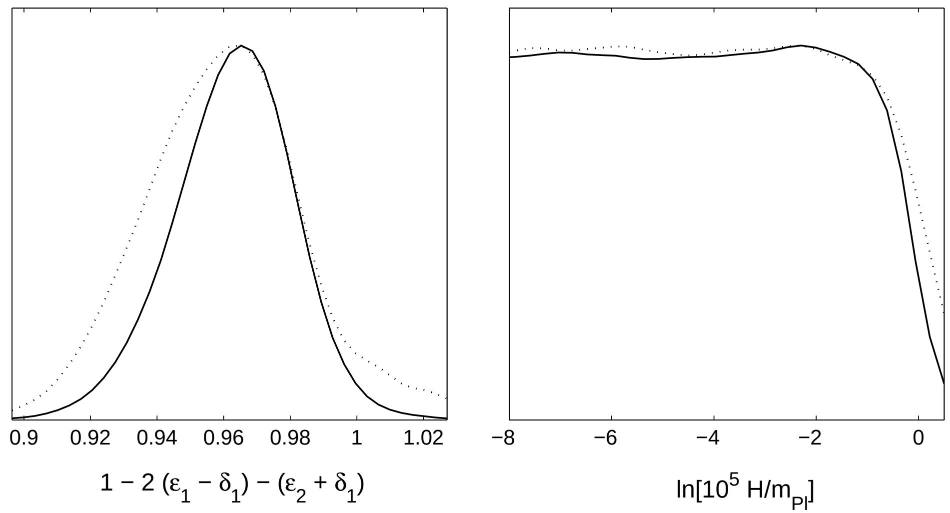
- 3+2 cosmological parameters: Ω_{dm}, Ω_{b}, H_0, τ, A_{SZ}
- 4 observable primordial parameters $(\mathcal{P}_\diamond, \epsilon_{1\diamond} - \delta_{1\diamond}, \epsilon_{2\diamond} + \delta_{1\diamond}, \epsilon_{1\diamond} c_{\mathrm{s}\diamond})$
  - Prior choice: $\ln$ on $\mathcal{P}_\diamond, \epsilon_{1\diamond} c_{\mathrm{s}\diamond}$, uniform on the flow parameters
  - Data used: WMAP5 + HST + Age
- Monte–Carlo–Markov–Chains analysis (`CAMB` + `COSMOMC`)



Posterior probability distributions for the cosmological parameters



Constraints on the k-inflationary parameters



- ϵ_1 alone is no longer constrained!
- CMB probes only amplitude, spectral index, tensor to scalar ratio

$$0.003 \leq 2(\epsilon_1 - \delta_1) + (\epsilon_2 + \delta_1) \leq 0.075$$

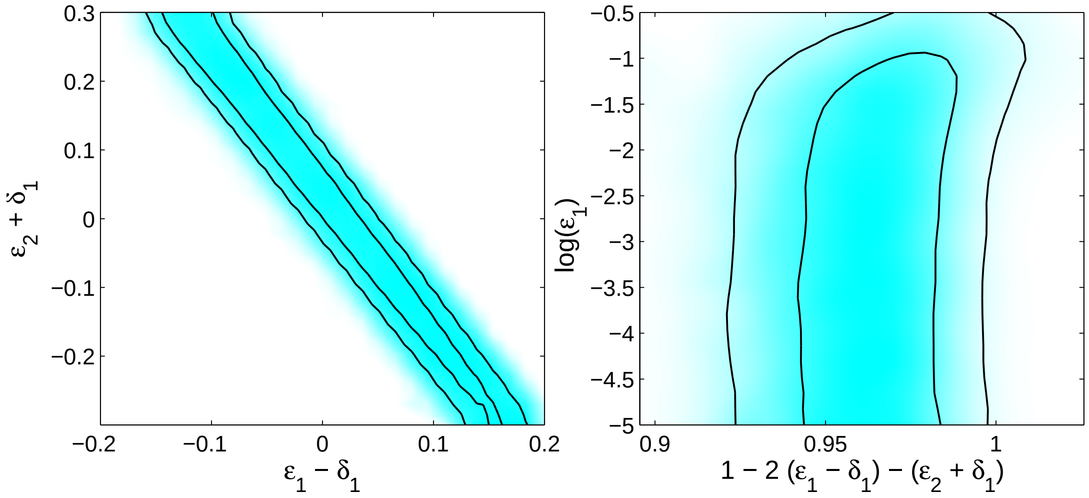
$$\ln\left(10^5 \frac{H_{\rm inf}}{m_{\rm Pl}}\right) \leq -0.59$$

$$\log\left(\epsilon_1 c_{\rm s}\right) \leq -2.3$$

- Non-gaussianity for DBI inflation (three points function) [Chen 07]

$$f_{\rm NL} = \frac{35}{108}\left(1 - \frac{1}{c_{\rm s}^2}\right) + \mathcal{O}(\epsilon_i, \delta_i)\,, \qquad -151 \leq f_{\rm NL} \leq 253 \quad (2\sigma \text{ WMAP5})$$

- Same data + non-gaussianity prior on $1 < 1/c_{\rm s}^2 < 467$ (2σ)



- Now (weakly) sensitive to the expansion acceleration: at two-sigma

$$\log \epsilon_1 \leq -1.1$$

Conclusion

- Non-minimal kinetic terms for primordial scalar fields
  - Natural in the context of brane
  - Allow inflation with non-flat potentials
- New dynamics $\Rightarrow$ use sound and Hubble flow parameters
  - Introduce a new hierarchy based on c_{s}
  - Primordial power spectra are sensitive to both
  - Corrected previous results: agreement only with [Kinney 08]
- Current constraints
  - CMB alone: almost all models are compatible (degeneracies)
  - One has to use non-gaussianity for DBI models

$$1 < 1/c_{\mathrm{s}}^2 < 467, \qquad \log \epsilon_1 < -1.1$$

DBI action from standard dynamics in higher dimensions

- 3-brane in a higher dimensional warped spacetime
  - Bulk: $\mathrm{d}s^2 = H_{AB}\mathrm{d}Y^A\mathrm{d}Y^B$ with

$$H_{AB}\mathrm{d}Y^A\mathrm{d}Y^B = h^{-1/2}(y)g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + h^{1/2}(y)G_{ab}(y)\mathrm{d}y^a\mathrm{d}y^b$$

  - 3-brane embedding $Y^A = X^A(x^\mu) \equiv \{x^\mu, \varphi^a(x^\mu)\}$
- Four-dimensional effective action
  - World sheet metric: $\gamma_{\mu\nu} = H_{AB}X^A_{,\mu}X^B_{,\nu}$
  - Poincaré invariance along the brane: Nambu-Goto

$$\mathcal{S} = -T_3 \int \sqrt{-\gamma}\,\mathrm{d}^4x = -T_3 \int \sqrt{-g}\frac{1}{h}\sqrt{\det\left(\delta^\nu_\mu + hG_{ab}\partial_\mu\varphi^a\partial^\nu\varphi^b\right)}\,\mathrm{d}x^4$$

- Brane motion in the bulk $\Rightarrow$ DBI inflation.
  - KKLMMT: $\mathcal{L} = -T(\varphi)\sqrt{1 + g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi/T(\varphi)} + \ldots$

DBI slow-roll trajectory

- DBI kinetic terms in a FLRW universe

$$\mathcal{L} = -\frac{T(\varphi)}{\gamma(\varphi,\dot{\varphi})} + T(\varphi) - V(\varphi), \qquad \gamma = \frac{1}{\sqrt{1-\dot{\varphi}^2/T(\varphi)}}$$

- DBI–Friedmann–Lemaître equations
  - In terms of the Hubble flow functions ϵ_n

$$\dot{\varphi} = -\frac{2}{\gamma}\frac{\mathrm{d}H}{\mathrm{d}\varphi}, \quad \epsilon_1 = \frac{2}{\gamma}\frac{1}{H^2}\left(\frac{\mathrm{d}H}{\mathrm{d}\varphi}\right)^2, \quad 3H^2 = \frac{V}{1-\dfrac{2\gamma}{3(\gamma+1)}\epsilon_1}$$

  - Assuming only $\epsilon_1 \ll 1 \Rightarrow H^2 \simeq V/3$

$$N(\varphi) = \mp\int_{\varphi_{\mathrm{ini}}}^{\varphi}\sqrt{\left(\frac{V}{V_{,\psi}}\right)^2 + \frac{1}{3}\frac{V}{T}}\,\mathrm{d}\psi$$