Gravitatational Reheating in Quintessential Inflation

Stefano Scopel

SEOUL NATIONAL UNIVERSITY

TAUP 2009, Rome, July 1st-5th, 2009

TAUP 2009

(based on 0904.0675, E.J. Chun, S.S. and I. Zaballa)

Acceleration in Cosmology

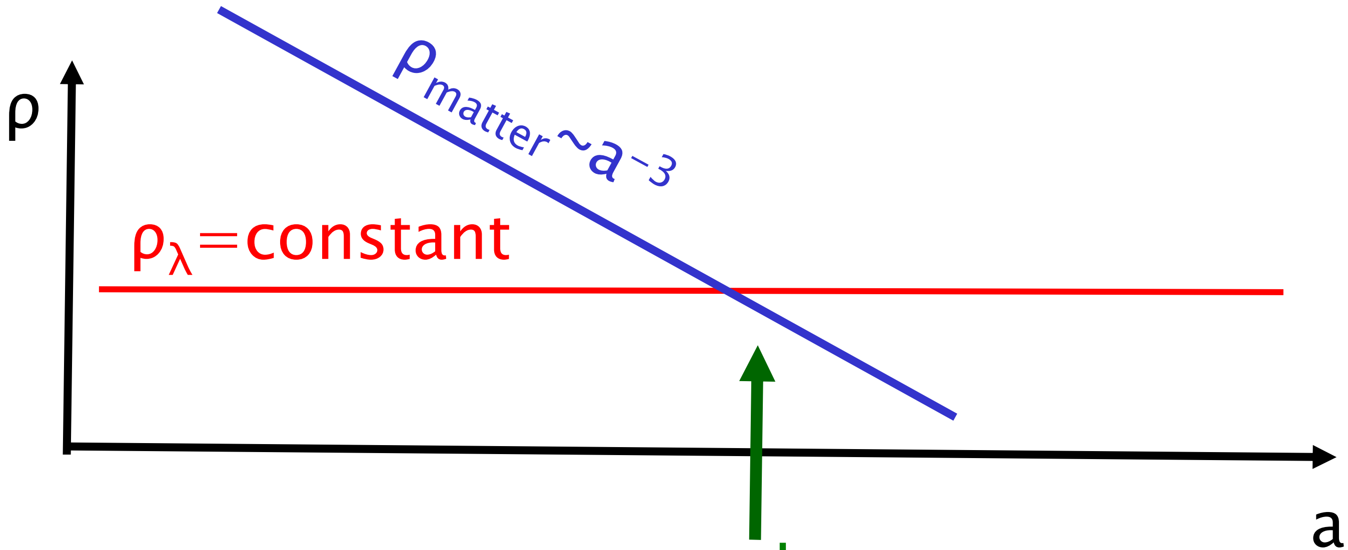
the Universe underwent a period of accelerated expansion with negative pressure at least twice in it history:

$$\begin{cases} \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{8\pi G}{3}\rho \\ \dot{\rho} + 3H(p+\rho) = 0 \\ \frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) \end{cases}$$

* if $p < -\rho/3 \Rightarrow \ddot{R} > 0$

- in it early stage during inflation → inflaton
- at the present day → quintessence

Today Ω_λ and Ω_{matter} are of the same order



Coincidence problem: WHY NOW?

$$w(z) = w_0 + w_a \frac{z}{1+z}$$, data seem to point to $w_0+w_a=0$, vacuum domination seems quite a recent event…)

quintessence field instead of cosmological constant can explain DE~DM more naturally through tracking solutions

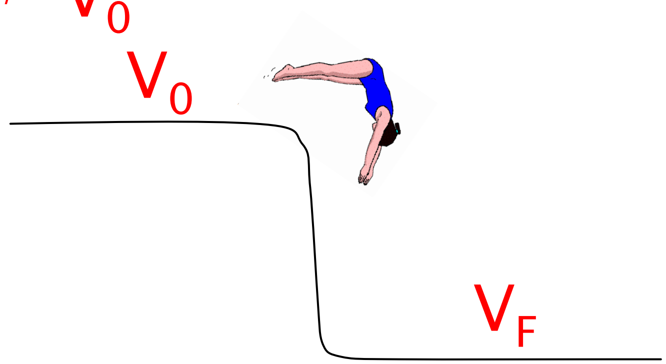
- inflation → inflaton
- accelerated expansion today → quintessence

} same field?

inflaton=quintessence → quintessential inflation

In quintessential inflation two main qualitative properties emerge:

1. the quintessence potential $V(\phi)$ needs to account for the large mismatch between the inflationary plateau at the beginning of the ϕ field evolution:
   $V_0 \sim m_P = 2.4 \times 10^{18}$ GeV
   and the tiny scale of the quintessential tail:
   $V_F \sim 10^{-121/4} V_0$



"deep dive" of the quintessence field at the end of inflation
→ kinetic energy dominates ("KINATION")

2. the inflaton does not decay and is still present today to account for DE → standard reheating mechanism is not at work

   If not from the inflaton decay, where does the entropy of the Universe come from???

Cosmological behaviour of kination

the energy-momentum tensor of quintessence :

$$T_{\mu\nu} = \partial_\mu \phi \frac{\partial \mathcal{L}}{\partial \partial^\nu \phi} - g_{\mu\nu} \mathcal{L}$$

equation of state:

$$w \equiv \frac{p}{\rho} = \frac{\frac{\dot{\phi}^2}{2} - V(\phi)}{\frac{\dot{\phi}^2}{2} + V(\phi)}$$

if: $\frac{\dot{\phi}^2}{2} \gg V(\phi) \longrightarrow \boxed{w = 1}$

The energy density of the Universe scales as $\rho \propto a^{-3(1+w)}$, so:

$$\begin{cases} \rho_{rad} \propto a^{-4} & \text{(radiation)} \\ \rho_{rad} \propto a^{-3} & \text{(matter)} \\ \rho_{kin} \propto a^{-6} & \text{(kination)} \end{cases}$$

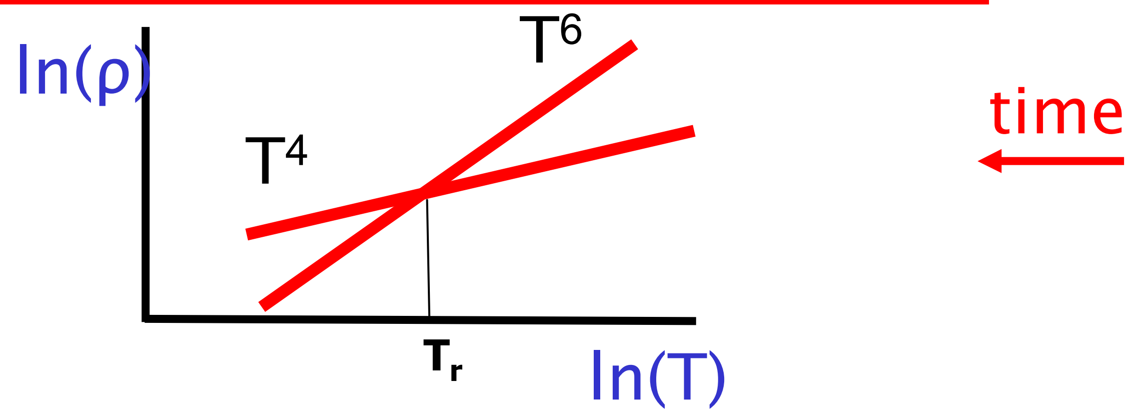
we know that radiation must dominate at the time of nucleosynthesis, however we have no observational constraint at earlier times. So setting T_r as the kination-radiation equality temperature for which:

$$\rho_{kin}(T_r) = \rho_{rad}(T_r)$$

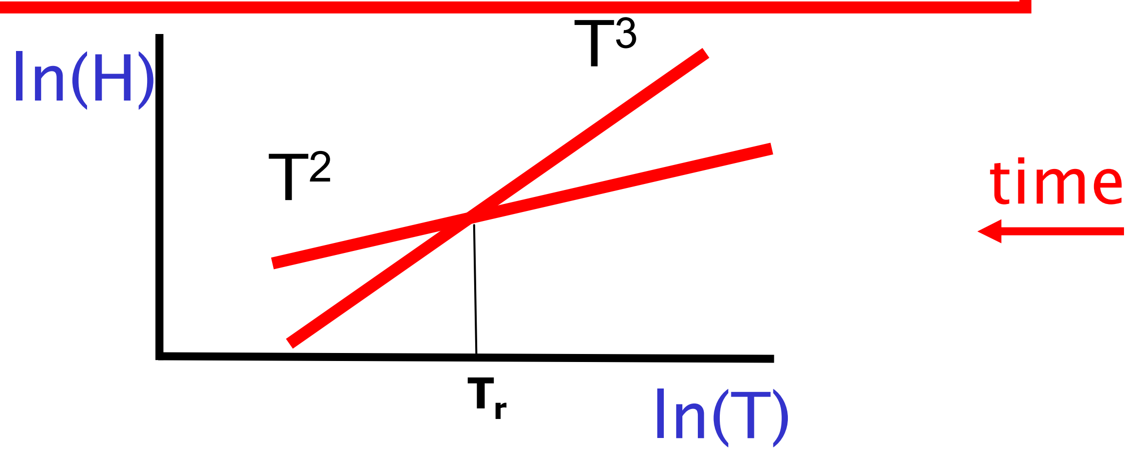
T_r is in general a free parameter, with the only bound:

$$T_r \gtrsim 1 \ \text{MeV}$$

$$\rho(T) = \frac{\pi^2}{30} g_* T^4 \left(1 + \frac{g_*}{g_{*r}} \left(\frac{T}{T_r} \right)^2 \right)$$



$$H(T) = 1.66 \sqrt{g_*} \frac{T^2}{m_{pl}} \sqrt{1 + \frac{g_*}{g_{*r}} \left(\frac{T}{T_r} \right)^2}$$



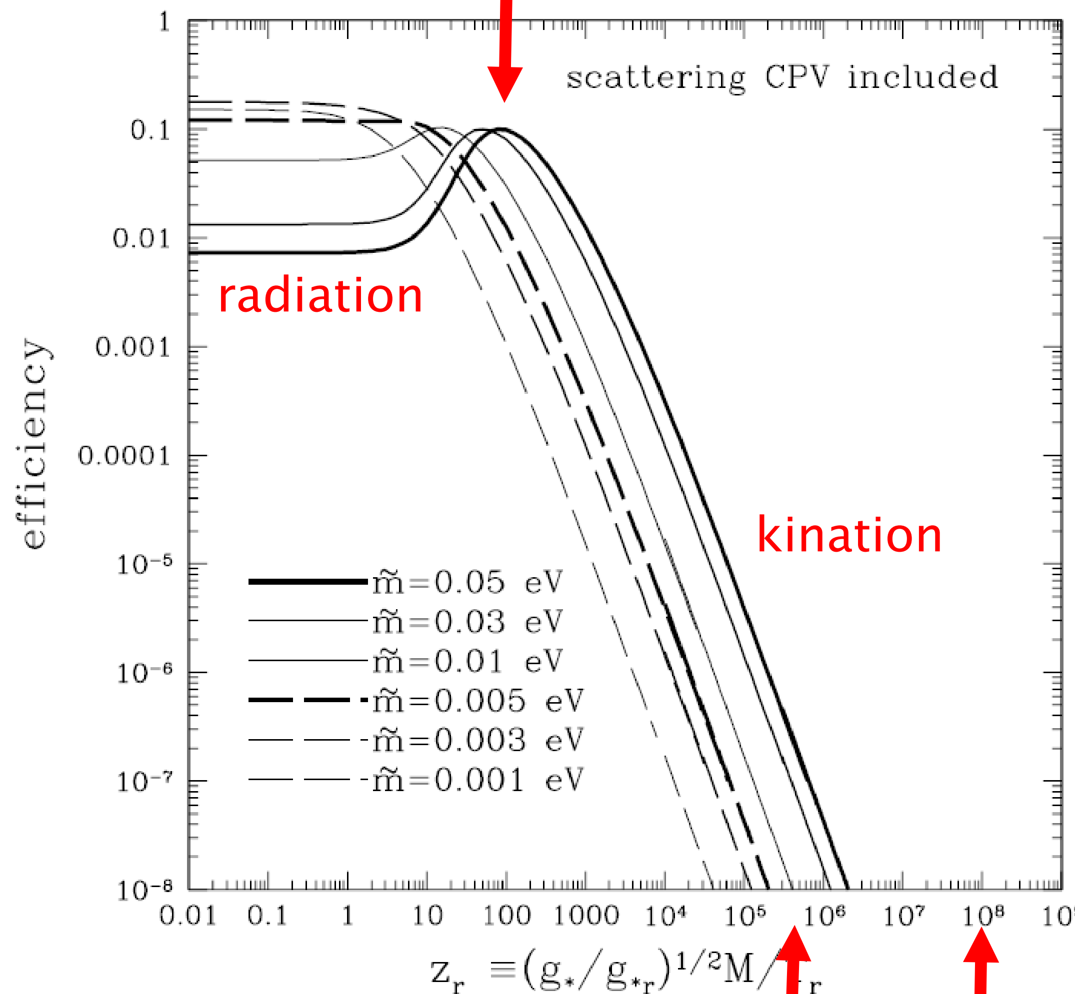
Impact of kination on thermal leptogenesis (E.J. Chun, S.S., JCAP10(2007)011)

efficiency η vs. z_r

- smooth transition from radiation dominance ($z_r<1$) to kination dominance ($z_r>1$)
- strong supression of the efficiency if $z_r>>1$
- increased efficiency for $1<z_r<100$ if m>0.01 eV

$$z_r \equiv \sqrt{\frac{g_*}{g_{*r}}} M/T_r$$

M=RH neutrino mass



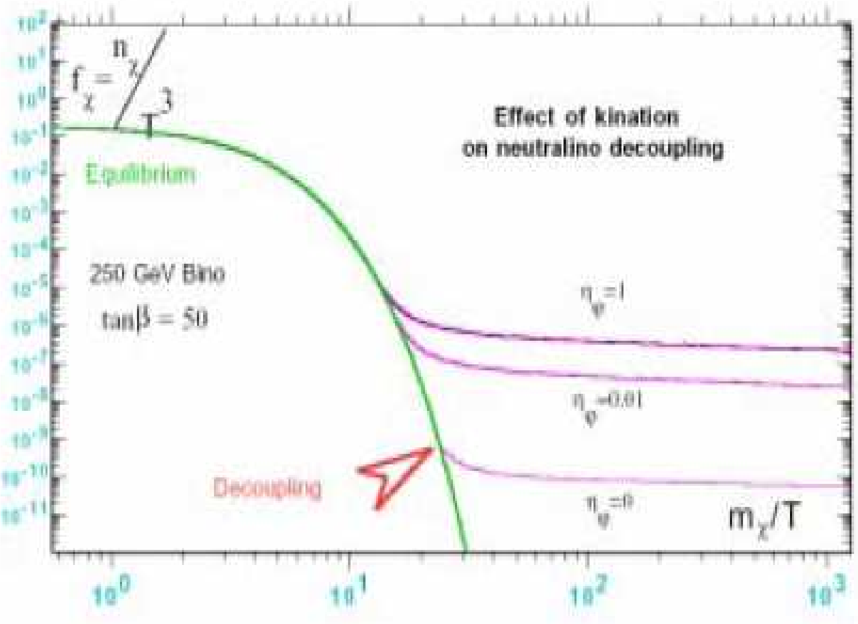
upper bound $z_r<9 \times 10^5$ ($\tilde{m}$/0.05 eV)

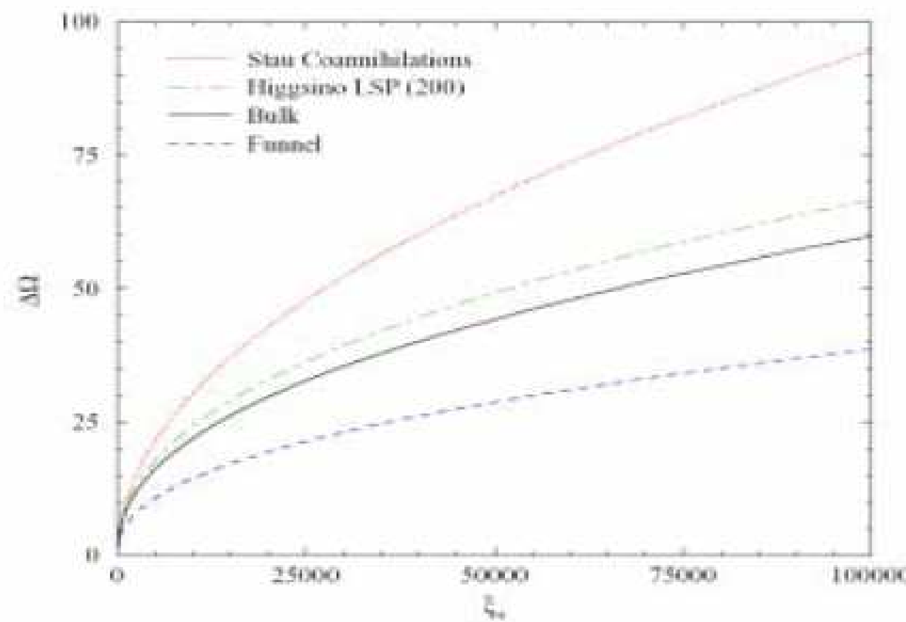
T_r=1 MeV→z_r~10^8

Impact on Dark Matter physics

During kination the Universe expands
faster than during radiation domination

a thermal Cold Dark Matter particle
decouples earlier and its relic density is enhanced





Salati, PLB571,121 $\eta_\phi \equiv \frac{\rho_\phi}{\rho_r}\Big|_{T_{NS}}$ $\xi_\phi \equiv \frac{\rho_\phi}{\rho_r}\Big|_{T_\chi}$ Profumo-Ullio, 0309220 the bottom line: the value of the temperature T_r for which kination and radiation give the same contribution to the energy density can have important phenomenological consequences for Dark Matter or Leptogenesis

what is the value of T_r in models of quintessential inflation?

Gravitational reheating

L. H. Ford, PRD35, 2955 (1987); Y. B. Zeldovich and A. A. Starobinsky, Sov. Phys. JETP34, 1159 (1972); N. D.Birrell, P.C.W. Davies and L. H. Ford, J.Phys. A13, 961 (1980).

particle creation ("field excitation") due to the changing spacetime metric at the end of the inflationary era

Action of scalar field:

$$S = \int d^4x \sqrt{|g|} \frac{1}{2} \left(g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - (m^2 + \xi R) \Phi^2 \right)$$

R=Ricci scalar=6 a''/a^3

in the background of a spatially flat Robertson–Walker metric:

$$ds^2 = a^2(\eta)(d\eta^2 - dx^i dx^i)$$

$$d\eta = \frac{dt}{a(t)}$$ =conformal time

spatial translation symmetry allows to separate spatial and time dependence of the field as:

$$\phi_k(\vec{x}, t) = \rho_k(t) e^{i\vec{k}\cdot\vec{x}}$$

factoring out the scale factor:

$$\chi_k \equiv \rho_k / a$$

one gets the equation of motion in conformal time:

$$\chi_k'' + \omega_k^2(\eta)\chi_k = 0$$

$$\text{where} \quad \omega_k^2(\eta) = k^2 + V = k^2 - \frac{a''}{a}$$

('=derivative with respect to conformal time)

assuming adiabatic conditions (ω_k=constant, i.e. V<<k) for $\eta \rightarrow \pm\infty$ the field can be expanded in terms of a complete set of positive-frequency solutions in the past and in the future f_i and F_i.

$$\phi = \sum_j \left(a_j f_j + a_j^\dagger f_j^* \right) = \sum_j \left(b_j F_j + b_j^\dagger F_j^* \right)$$

the operators a_j and b_j annihilate the in and out vacua and are connected by the Bogolubov transformations:

$$a_j = \sum_k \left(\alpha^*_{jk} b_k - \beta^*_{jk} b^\dagger_k\right)$$
$$b_k = \sum_j \left(\alpha_{jk} a_j + \beta^*_{jk} a^\dagger_j\right)$$

assuming that at $\eta=-\infty$ no particles are present the initial vacuum $|0>_{in}$ (which in the Heisenberg picture is the state of the system for *all* the time) is annihilated by the a_j operators:

$$a_j|0>_{in}=0$$

However the physical number operator that counts particles in the out region is

$$< N_k >=_{in}< 0|b^\dagger_k b_k|0 >_{in}= \sum_j |\beta_{jk}|^2 \neq 0$$

→ particle creation proportional to the negative-frequency content of the out states with the boundary condition that the in states contain only negative frequencies

Solve differential equation:

$$\chi_k'' + \omega_k^2(\eta)\chi_k = 0$$

$$\omega_k^2(\eta) = k^2 + V = k^2$$

(harmonic oscillator with time-dependent frequency)

"potential" V=-a''/a, a=scale factor

with boundary conditions:

$$\begin{aligned} \chi_k(\eta \to -\infty) &= \frac{e^{-ik\eta}}{\sqrt{2k}} \\ \chi_k(\eta \to +\infty) &= \frac{1}{\sqrt{2k}}\left(\alpha_k e^{-ik\eta} + \beta_k e^{ik\eta}\right) \end{aligned}$$

$(k \equiv |\vec{k}|)$

Wronskian condition $\chi_k \chi_k^{*\prime} - \chi_k^* \chi_k' = i$ keeps the correct normalization of states and implies the additional relation:

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

setting:

$$\chi_k = r_k e^{i\phi_k}$$

the problem reduces to solving the single second-order differential equation:

$$r_k'' + \left(\omega_k^2(\eta) - \frac{1}{4r_k^4}\right) r_k = 0$$

with boundary conditions:

$$r(\eta \to -\infty) = \frac{1}{\sqrt{2k}}$$

$$r'(\eta \to -\infty) = 0$$

$$|\beta_k|^2 = \frac{k^2 r_k^2 + (r_k')^2 + \frac{1}{4r_k^2} - k}{2k} \quad (\eta \to \infty)$$

energy density produced:

$$\rho = \frac{1}{(2\pi a)^3 a} \int d^3\vec{k}\, k |\beta_k|^2$$

$T_{RH} \sim k\, H_e/2\pi$, Hawking radiation

Typically the energy density of radiation produced through this process is very small. However, in quintessential inflation at the end of the inflationary period the energy density of the Universe is dominated by kination, which dilutes faster than radiation. So eventually at some temperature $T_* = T_r > 1$ MeV (to preserve nucleosynthesis) radiation dominates

gravitational reheating already studied by Ford, PRD35,2955(1987) in the case of inflation-radiation transition. That result widely used also in papers discussing quintessential inflation.

generalization to inflation-kination transition (E.J. Chun, S.S. I. Zaballa, 0904.0675).

Two parametrizations for the inflation–kination transition:

Parametrization 1

$$f(x) = \begin{cases} 1/x^2 & \text{for } x < -1 \quad \text{(inflation)} \\ a_0 + a_1 x + a_2 x^2 + a_3 x^3 & \text{for } -1 < x < x_0 - 1 \quad \text{(transition)} \\ b_0 + b_1 x & \text{for } x > x_0 - 1 \quad \text{(kination)} \end{cases}$$

$$a^2(\eta) \equiv f(x)$$

$$x_0 = H_0 \Delta\eta$$

=duration of the transition ($x_0 \to 0$ known to lead to ultraviolet divergence)

H_0=Hubble constant during inflation

coefficients $a_i(x_0)$ found in such a way that f(x) and f'(x) are continuous:

$$\begin{cases} a_0 = -\frac{1}{x_0} + 6, \quad a_1 = -\frac{3}{x_0} + 8, \quad a_2 = -\frac{3}{x_0} + 3, \quad a_3 = -\frac{1}{x_0} \\ b_0 = 3 + 3x_0 - x_0^2, \quad b_1 = 2 + 3x_0. \end{cases}$$

Parametrization 2

$$w(y) = \frac{w_f - 1}{2} + \frac{w_f + 1}{2} \tanh\left(\frac{2y}{y_0}\right)$$

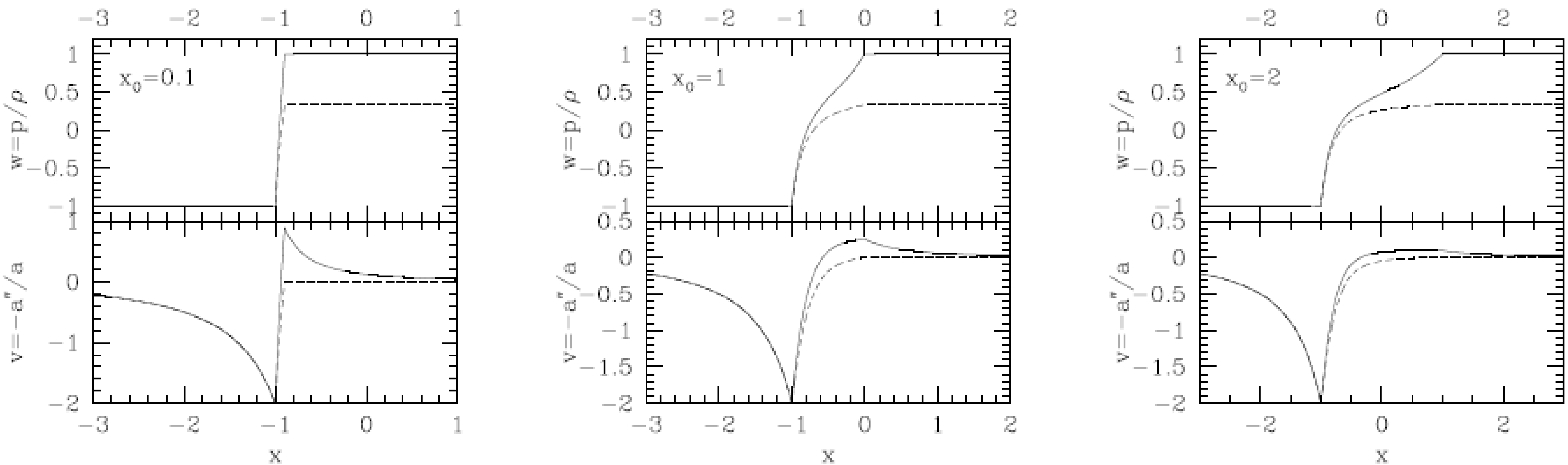
$w_f=1$ (kination)

$$y \equiv \log(a) + c$$ (number of e-foldings during inflation)

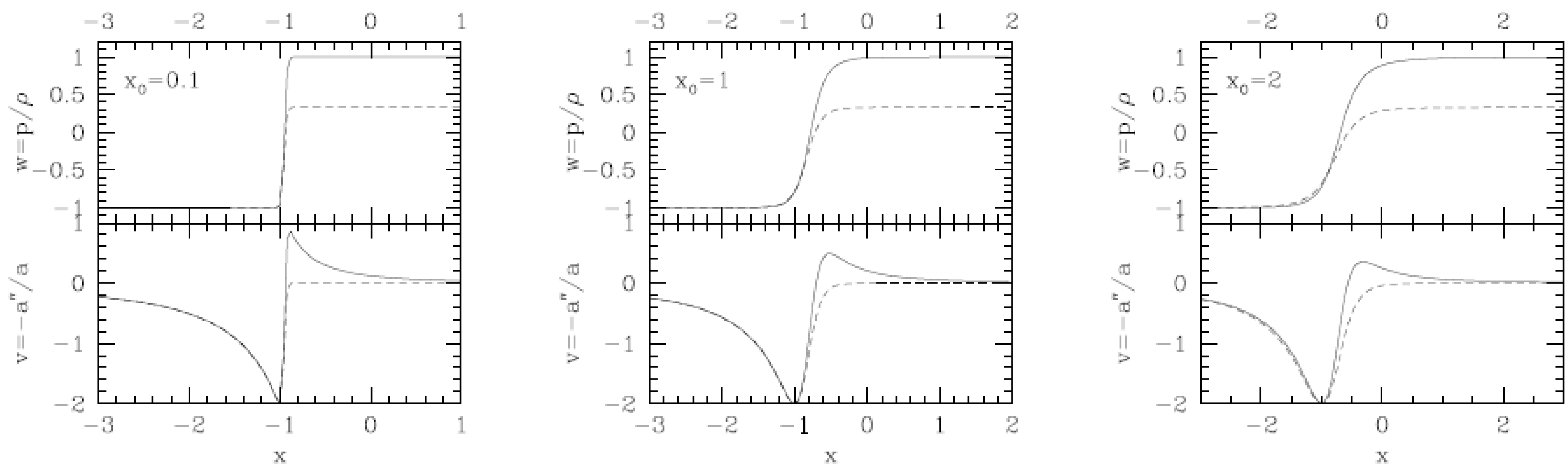
N.B. absolute normalization of the scale factor is arbitrary. We fix the constant c in such a way that function V=-a''/a has the same behaviour in both parametrization (actually, in both parametrizations the minimum of V is for $\eta=-H_0^{-1}$ and in the minimum the normalization of V is the same: $V(-H_0^{-1})=-2$)

Function V and equation of state w as a function of $x = H_0 \eta$

First parametrization



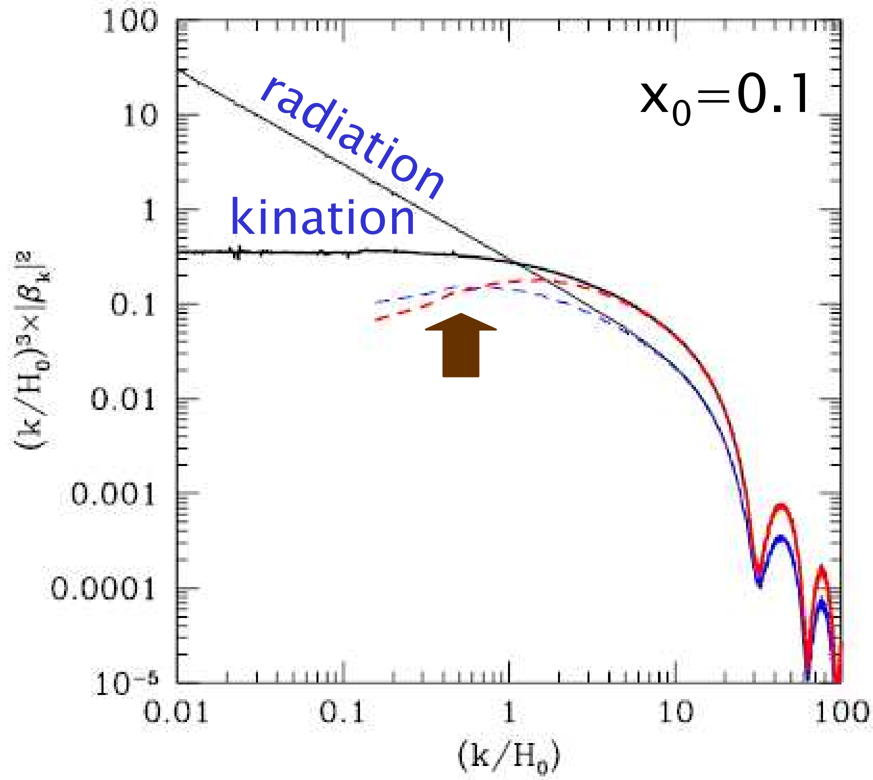
Second parametrization



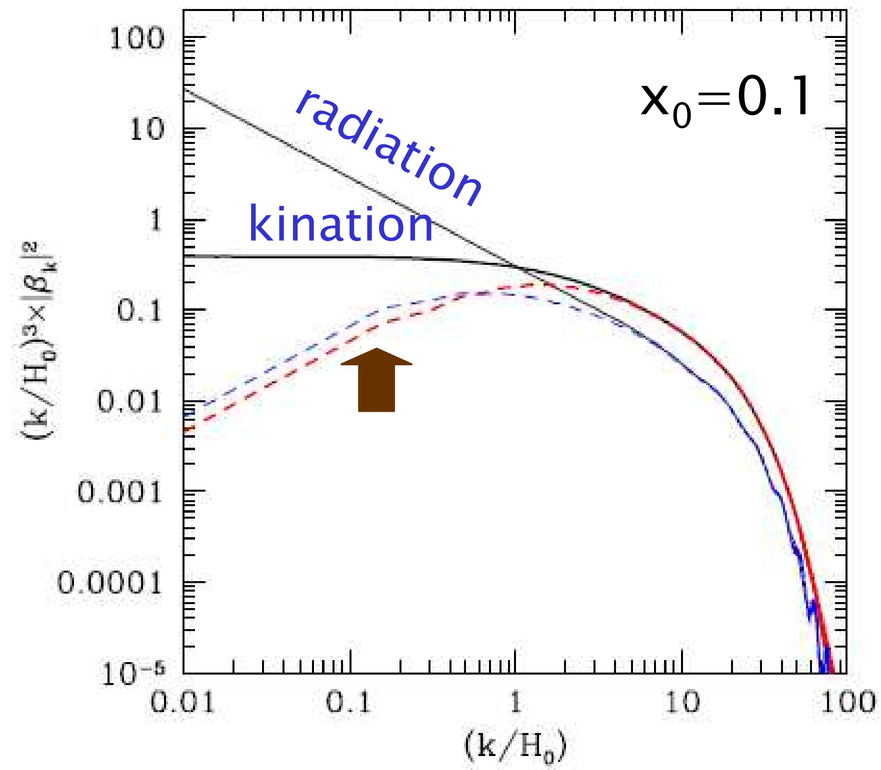
Numerical results

$$\rho = \frac{1}{(2\pi a)^3 a} \int d^3\vec{k}\, k |\beta_k|^2$$

first parametrization



second parametrization



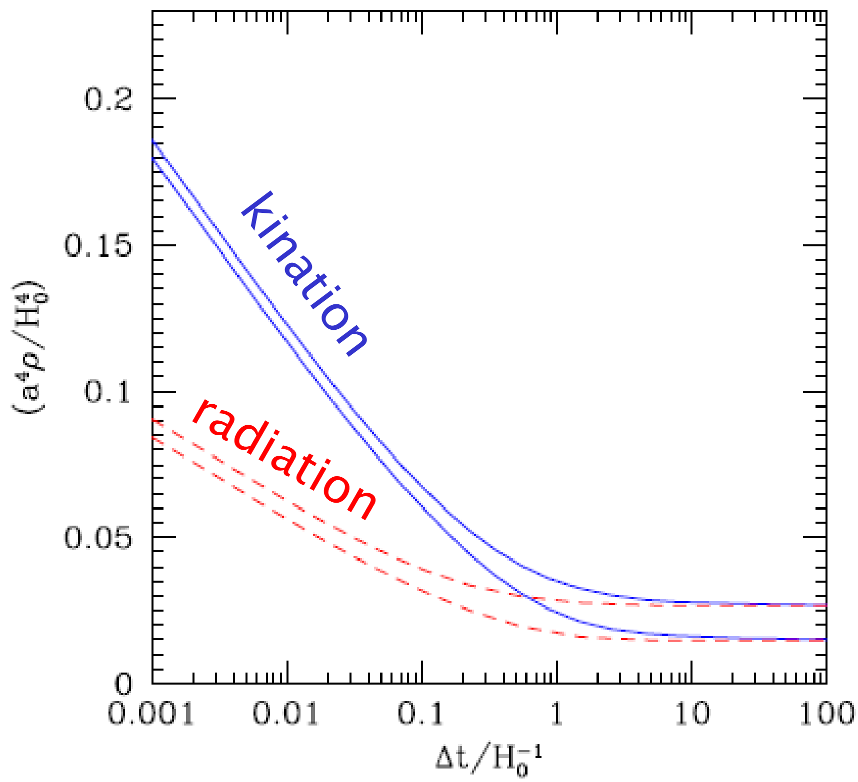
approximate results in the adiabatic limit, k>>V:

$$\beta_k \simeq -\frac{i}{2k} \int_{-\infty}^{+\infty} e^{-2ik\eta} V(\eta)\, d\eta$$

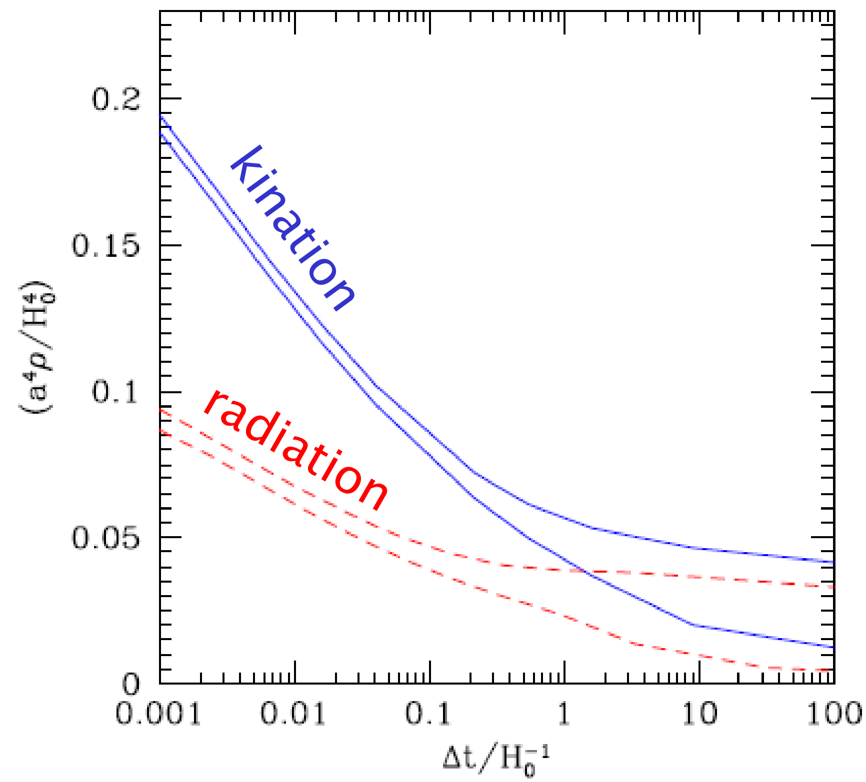
Numerical results

Energy density per scalar degree of freedom produced by gravitational reheating vs. duration of the transition expressed in cosmic time

first parametrization



second parametrization



- enhancement when $\Delta t << H_0^{-1}$ more pronounced in the case of inflation-kination transition (second peak in function V)

- plateau for $\Delta t >> H_0^{-1}$

Radiation-kination equality

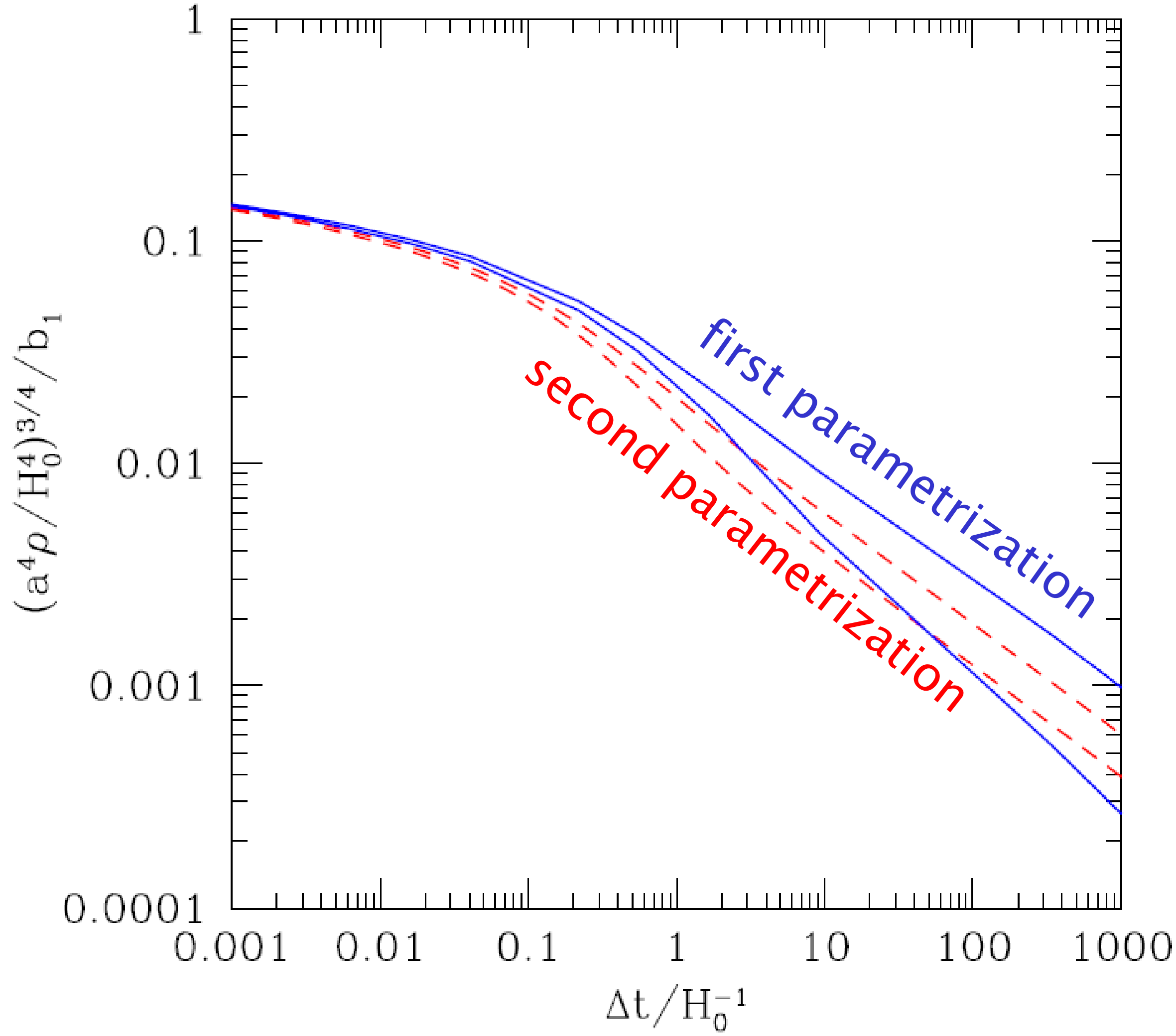
$$T_* = \frac{2\times 30^{\frac{1}{4}}}{\sqrt{3\pi}}\frac{g_I}{g_*^{\frac{1}{2}}}\frac{I^{\frac{3}{4}}}{b_1}\frac{H_0^2}{m_P}$$

T_I= temperature at some late time when the adiabatic condition is safely verified
g_I=# of relativistic degrees of freedom at T_I
$I=a^4\rho/H_0{}^4$
b_1= normalization of Hubble parameter during kination:

$$H/H_0=b_1/(2a^3)$$

Numerical results

combination $I^{3/4}/b_1$ vs. duration of the transition in cosmic time



Asymptotic behaviours

$$\frac{I^{\frac{3}{4}}}{b_1} \sim \begin{cases} 0.06\,[\ln \frac{1}{H_0\Delta t}]^{\frac{3}{4}} & \text{for} \quad \Delta t \to 0 \\ {\substack{0.02 \\ 0.026}}\,(H_0\Delta t)^{\substack{-0.5 \\ -0.48}} & \text{for} \quad \Delta t \to \infty \end{cases}$$

$$T_* \sim \frac{g_I}{\sqrt{g_*}}\frac{H_0^2}{m_P} \begin{cases} 0.09\,[\ln \frac{1}{H_0\Delta t}]^{\frac{3}{4}} & \text{for} \quad \Delta t \to 0 \\ {\substack{0.03 \\ 0.039}}\,(H_0\Delta t)^{\substack{-0.5 \\ -0.48}} & \text{for} \quad \Delta t \to \infty \end{cases}$$

Conclusions

- The kination-radiation equality temperature T_* depends on the inflationary scale H_0 and on the transition time Δt
- The dependence on Δt is logarithmic at small Δt and $\sim(\Delta t)^{-0.5}$ at large Δt
- Compared to the standard situation of a transition between inflation and radiation, gravitational reheating with kination is enhanced by a factor of ~2 at small Δt, while the two effects are similar at large Δt
- The epoch of particle production is confined to a short interval of time close to the end of inflation even in the case $\Delta t >> H_0^{-1}$
- Depending on the particular choice of the inflationary potential $1\ \mathrm{MeV} \lesssim T_* \lesssim 10^{10}\ \mathrm{GeV}$ → Dark Matter relic density and leptogenesis can be affected