An Analysis of Cosmic Neutrino
Flavor Composition at Source and Neutrino Mixing Parameters

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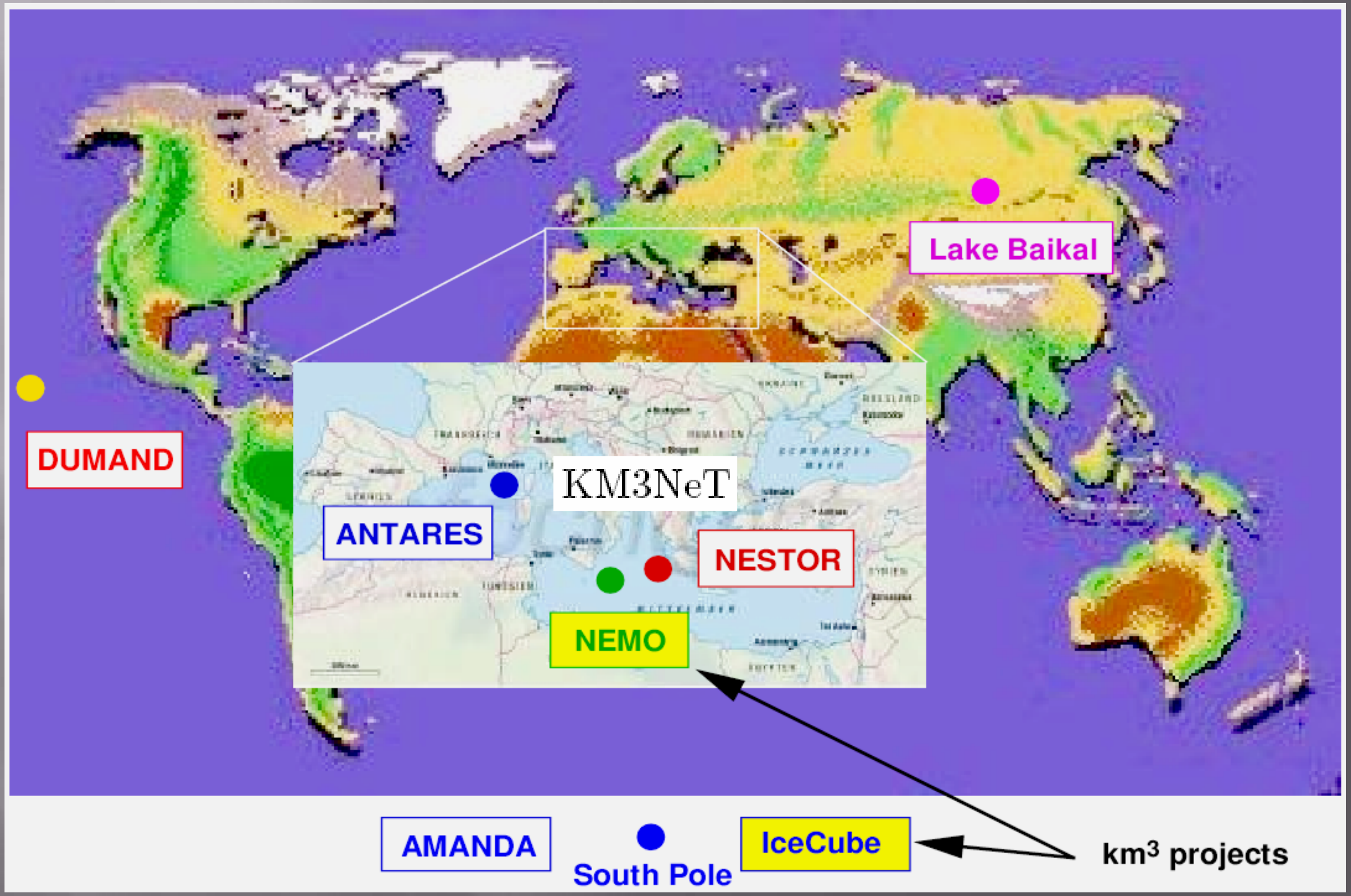
A. E. and Yasaman Farzan; arXiv:0905.0259;   to be published NPB

OUTLINE

- A brief introduction to neutrino telescopes
- Flavor identification at neutrino telescopes
- Uncertainties in the inputs
- Determination of CP-violating phase δ
- Flavor composition at the source

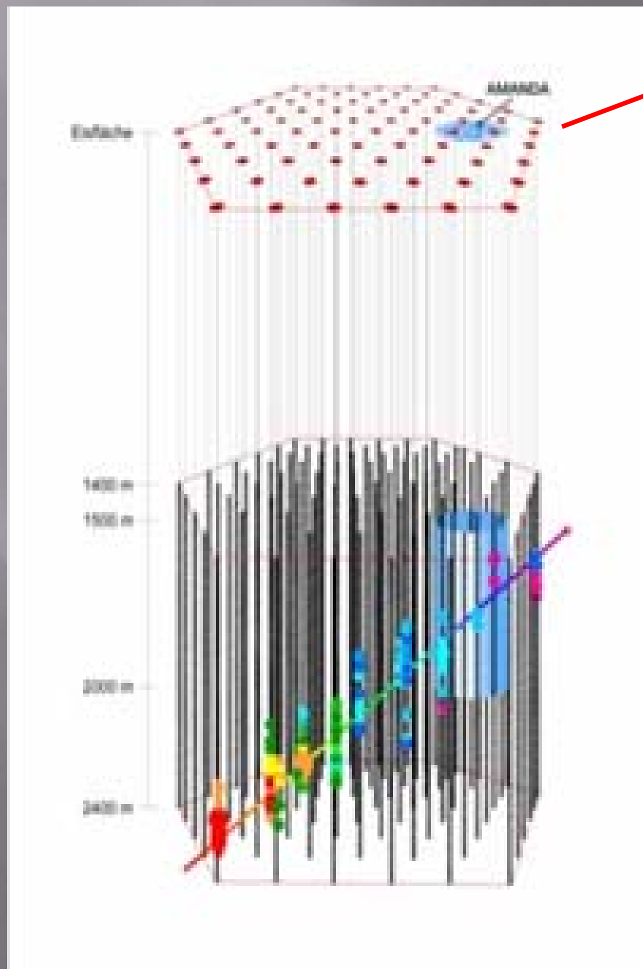
Neutrino Telescopes World Map



Current upper limit on the diffuse flux of neutrinos from AMANDA experiment

$$E_\nu^2 \frac{dF_\nu}{dE_\nu} \leq 8.2 \quad \text{GeV cm}^{-2}\ \text{sr}^{-1}\ \text{yr}^{-1}$$

Depth of the ice at the IceCube site ~ 2850 m



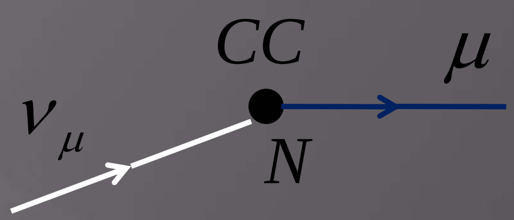
Ice top

PMTs loaded from 1450 m to 2450 m

Flavor Identification

IceCube can distinguish two types of events

- Muon-track events
- CC interaction of ν_μ



Area

Avogadro's number

Density

$$\rho A N_A \iint R_\mu(E_\mu, E^\mu_{th}) \frac{dF_{\nu_\mu}}{dE_{\nu_\mu}} \frac{d\sigma^{CC}}{dE_\mu} dE_\mu dE_{\nu_\mu} + [\text{particle} \rightarrow \text{antiparticle}]$$

Muon Range

$$R_\mu(E_1, E_2) = (2.6\ \text{Km}) \ln \left[\frac{2 + 4.2 \times 10^{-3} E_1}{2 + 4.2 \times 10^{-3} E_2} \right]$$

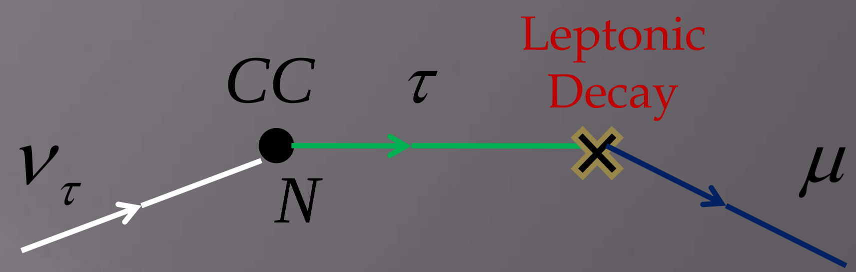
The energy threshold for the detection of muon neutrinos

$E_{th} \sim 100$ GeV

To avoid considering neutrino absorption at Earth we set

$E_\upsilon^{cut} = 100$ TeV

- CC interaction of ν_τ



The effect of this event can be as large as s^2_{13}

$$B\rho A N_A \int^{E_{cut}} \int \int_{E^{\mu}_{th}} \frac{dF_{\nu_\tau}}{dE_{\nu_\tau}} \frac{d\sigma^{CC}}{dE_\tau} f\left(E_\tau, E_\mu\right) R_\mu(E_\mu, E^{\mu}_{th}) dE_\mu dE_\tau dE_{\nu_\tau} + (\nu_\tau \rightarrow \bar{\nu}_\tau)$$

$$B \equiv \mathrm{Br}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau) = 17.8\%$$

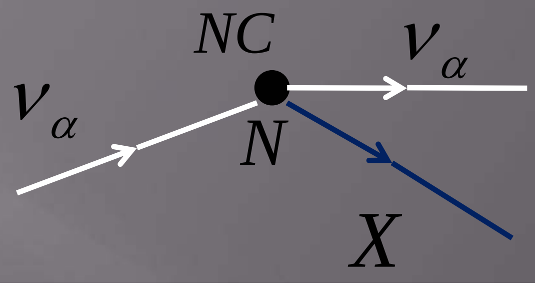
$$f(E_\tau, E_\mu) \equiv \frac{1}{\Gamma} \frac{d\Gamma(\tau(E_\tau) \rightarrow \mu(E_\mu)\bar{\nu}_\mu \nu_\tau)}{dE_\mu}$$

In the limit of high boost factor ($\gamma >> 1$ or $\beta \rightarrow 1$)

$$f(E_\tau, E_\mu) \simeq \frac{5}{3E_\tau} - \frac{3E_\mu^2}{E_\tau^3} + \frac{4E_\mu^3}{3E_\tau^4}$$

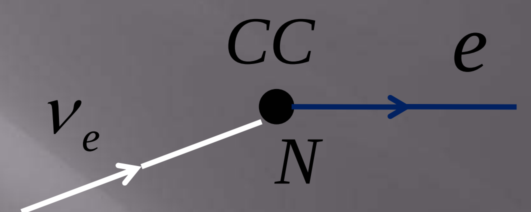
Shower-like events

- NC interaction of ν_α



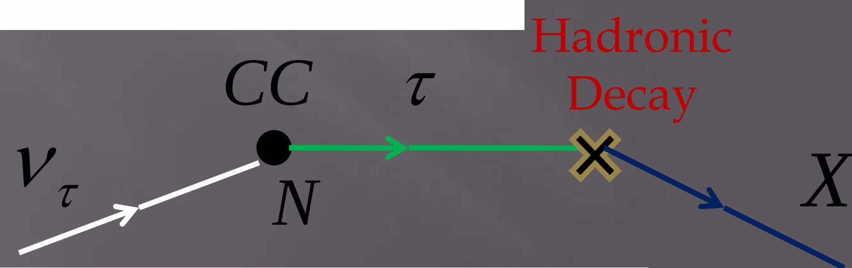
$$\sum_{l=e,\mu,\tau} \rho A L N_A \left[\int^{E_{cut}} \frac{dF_{\nu_l}}{dE_{\nu_l}} \sigma^{NC} dE_{\nu_l} + \int^{E_{cut}} \frac{dF_{\bar{\nu}_l}}{dE_{\bar{\nu}_l}} \bar{\sigma}^{NC} dE_{\bar{\nu}_l} \right]$$

- CC interaction of ν_e



$$\rho A L N_A \left[\int^{E_{cut}} \frac{dF_{\nu_e}}{dE_{\nu_e}} \sigma^{CC} dE_{\nu_e} + \int^{E_{cut}} \frac{dF_{\bar{\nu}_e}}{dE_{\bar{\nu}_e}} \bar{\sigma}^{CC} dE_{\bar{\nu}_e} \right]$$

- CC interaction of ν_τ



$$(1 - B) \rho A L N_A \left[\int^{E_{cut}} \frac{dF_{\nu_\tau}}{dE_{\nu_\tau}} \sigma^{CC} dE_{\nu_\tau} + \int^{E_{cut}} \frac{dF_{\bar{\nu}_\tau}}{dE_{\bar{\nu}_\tau}} \bar{\sigma}^{CC} dE_{\bar{\nu}_\tau} \right]$$

The quantity that can be measured in IceCube

$$R = \frac{\text{Number of Muon-track events}}{\text{Number of Shower-like events}}.$$

~ one order of magnitude lower than the present upper bound

With a neutrino flux $E_\nu^2 dF_\nu/dE_\nu = 0.25 \text{ GeV cm}^{-2} \text{ sr}^{-1} \text{ yr}^{-1}$

According to J. F. Beacom *et al.*, Phys. Rev. D 68 (2003)

A few hundreds of events in a couple of years

R can be measured by **7%** precision

Inputs and Uncertainties

Spectrum of the incoming flux of neutrinos :

$$\frac{dF_\nu}{dE_\nu} = N_\nu E_\nu^{-\alpha}$$

Normalization factor

Models of neutrino production (Fermi acceleration) predict $\alpha = 2$

But, considering non-linear effects $1 < \alpha < 3$

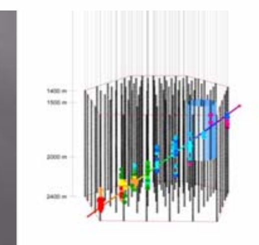
Can be measured by 10% precision in IceCube

J. F. Beacom *et al.*, Phys. Rev D68 (2003)

Flavor ratios :

Traveling distance L ~ 100 Mpc

$$E_\nu^2 dF_\nu/dE_\nu = 0.25 \text{ GeV cm}^{-2} \text{ sr}^{-1} \text{ yr}^{-1}$$



Production of neutrinos at the source with flavor ratio

$$w_e^0 : w_\mu^0 : w_\tau^0$$

1 : 2 : 0

At the Earth the flavor ratio will be

$$\sum_\alpha w_\alpha^0 P_{\alpha e} : \sum_\alpha w_\alpha^0 P_{\alpha\mu} : \sum_\alpha w_\alpha^0 P_{\alpha\tau}$$

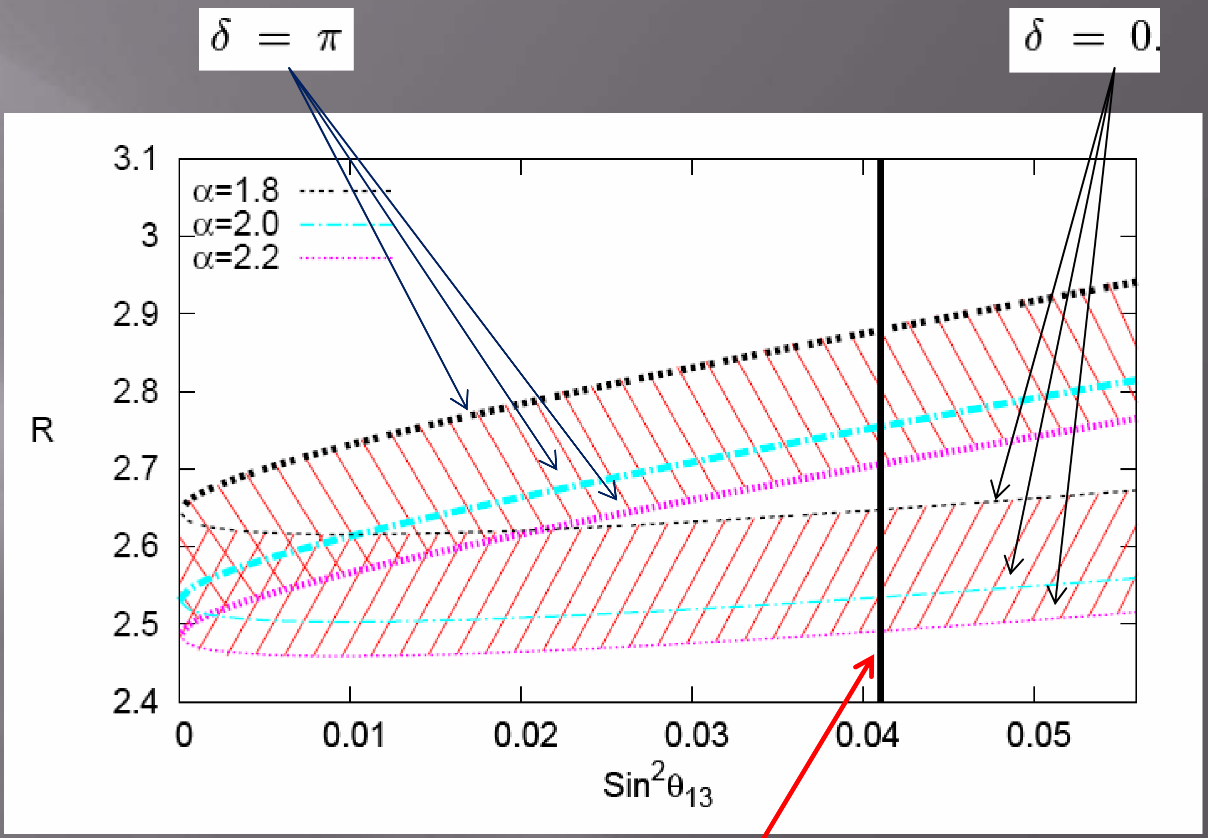
$$P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Sensitivity to $s_{13}{}^2$

$(\mathcal{N}_{\bar{\nu}_\mu} + \mathcal{N}_{\nu_\mu})/(\mathcal{N}_{\bar{\nu}_e} + \mathcal{N}_{\nu_e}) = 2$

$\mathcal{N}_{\bar{\nu}_e}/\mathcal{N}_{\nu_e} = 0.5$

θ_{12} and θ_{23} → Best fit values



The present bound on $s_{13}{}^2$ at 3σ

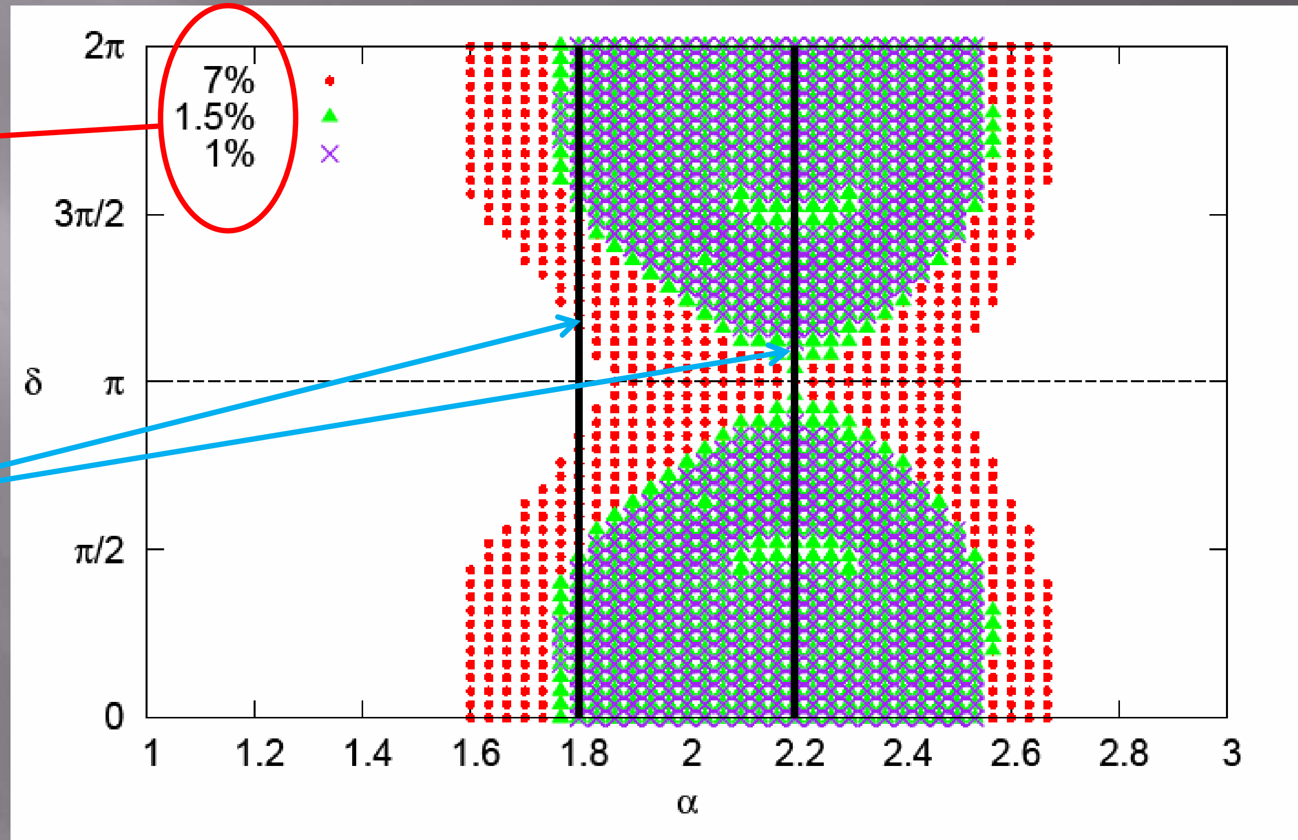
- ♠ *R* is more sensitive to $s_{13}{}^2$ in the case $\delta = \pi$ (~ 10 %)
- ♠ Even for $\delta = \pi$, the sensitivity of *R* is obscured by the 10 % uncertainty in α
- ♠ For $s_{13}{}^2 > 0.02$, it is possible to distinguish $\delta = 0$ from $\delta = \pi$

Determination of δ

$\delta = \pi/2$ & $w_e^0 : w_\mu^0 : w_\tau^0 = 1 : 2 : 0$

Uncertainty in the measurement of ***R***

10 % precision in the measurement of α with central value α = 2



- ♠ ***R*** depends on δ through cos δ , symmetry under δ → 2π − δ
- ♠ By measuring ***R*** with 7 % precision, δ cannot be constrained
- ♠ By measuring ***R*** with 1 % precision, regions around δ = π can be excluded

$\theta_{13} = 10^\circ$, near the present bound

~ 6 % uncertainty

~ 5 % uncertainty

$\sin^2\theta_{13} = 0.03^{+0.002}_{-0.002}$

$\sin^2\theta_{23} = 0.5^{+0.03}_{-0.03}$

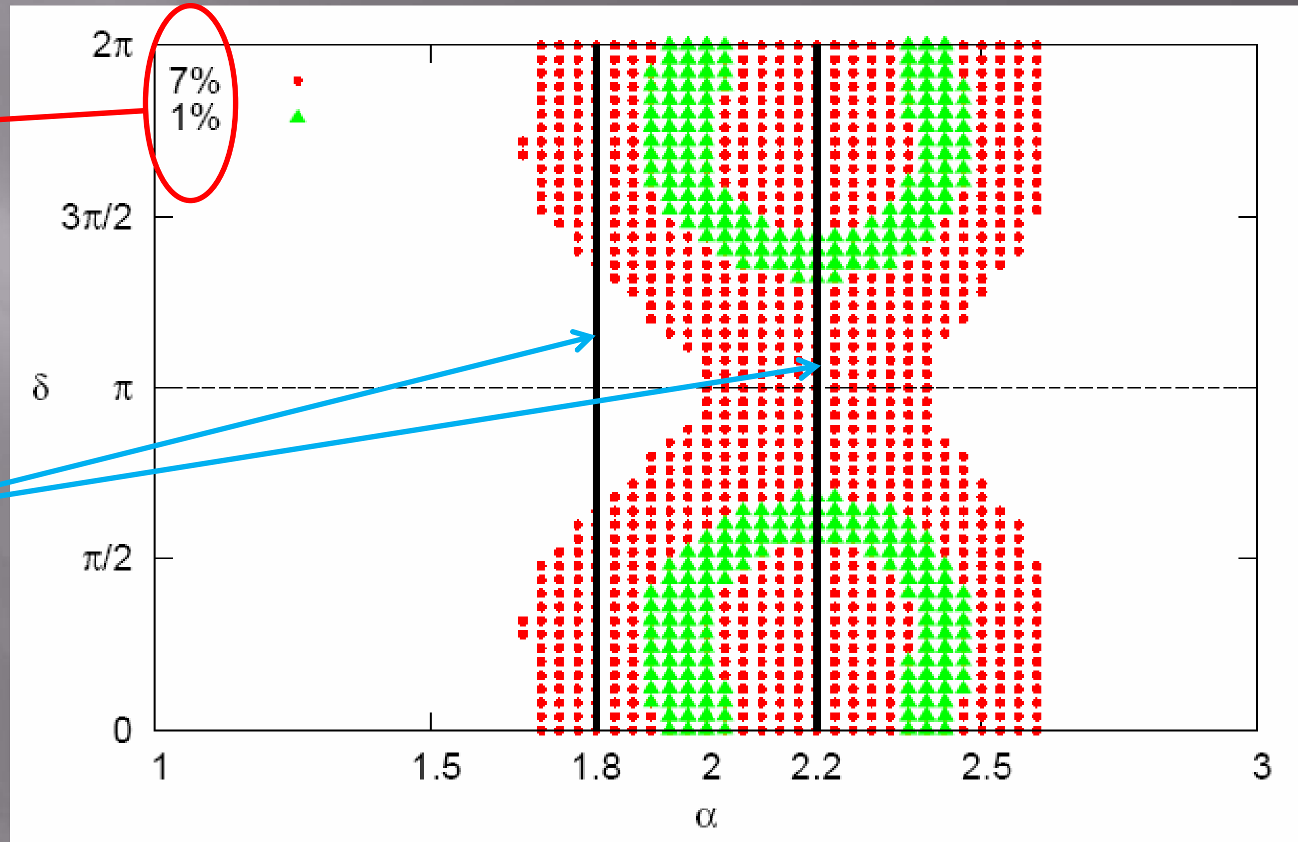
$\sin^2\theta_{12} = 0.32^{+0.02}_{-0.02}$

$\mathcal{N}_{\bar{\nu}_e}/\mathcal{N}_{\nu_e} \in (0, 1)$

Determination of δ

$\delta = \pi/2$ & $w_e^0 : w_\mu^0 : w_\tau^0 = 1 : 2 : 0$

Uncertainty in the measurement of *R*



10 % precision in the measurement of α with central value α = 2

- ♠ ***R*** is significantly sensitive to the uncertainty in θ_{23}
- ♠ Reducing the uncertainty in $\sin^2\theta_{23}$ from 6 % to 1 % removes a substantial part of spurious solutions

Fixed to their central values

1 % precision interval

$\sin^2\theta_{23} = 0.5^{+0.005}_{-0.005}$

$\sin^2\theta_{13} = 0.03$

$\sin^2\theta_{12} = 0.32$

$\mathcal{N}_{\bar\nu_e}/\mathcal{N}_{\nu_e} = 0.5$

Flavor Composition at the Source

Pion sources

$$\begin{aligned} \pi^- &\rightarrow \mu^- \bar{\nu}_\mu \\ \mu^- &\rightarrow e^- \nu_\mu \bar{\nu}_e \end{aligned}$$

Main mechanism of neutrino production

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ \nu_\mu \\ \mu^+ &\rightarrow e^+ \bar{\nu}_\mu \nu_e \end{aligned}$$

$$w_e^0 : w_\mu^0 : w_\tau^0 = 0.5 : 1 : 0$$

$$w_e^0 : w_\mu^0 : w_\tau^0 = 0 : 1 : 0$$ → Stopped-muon source

$$w_e^0 : w_\mu^0 : w_\tau^0 = 1 : 0 : 0$$ → Neutron source

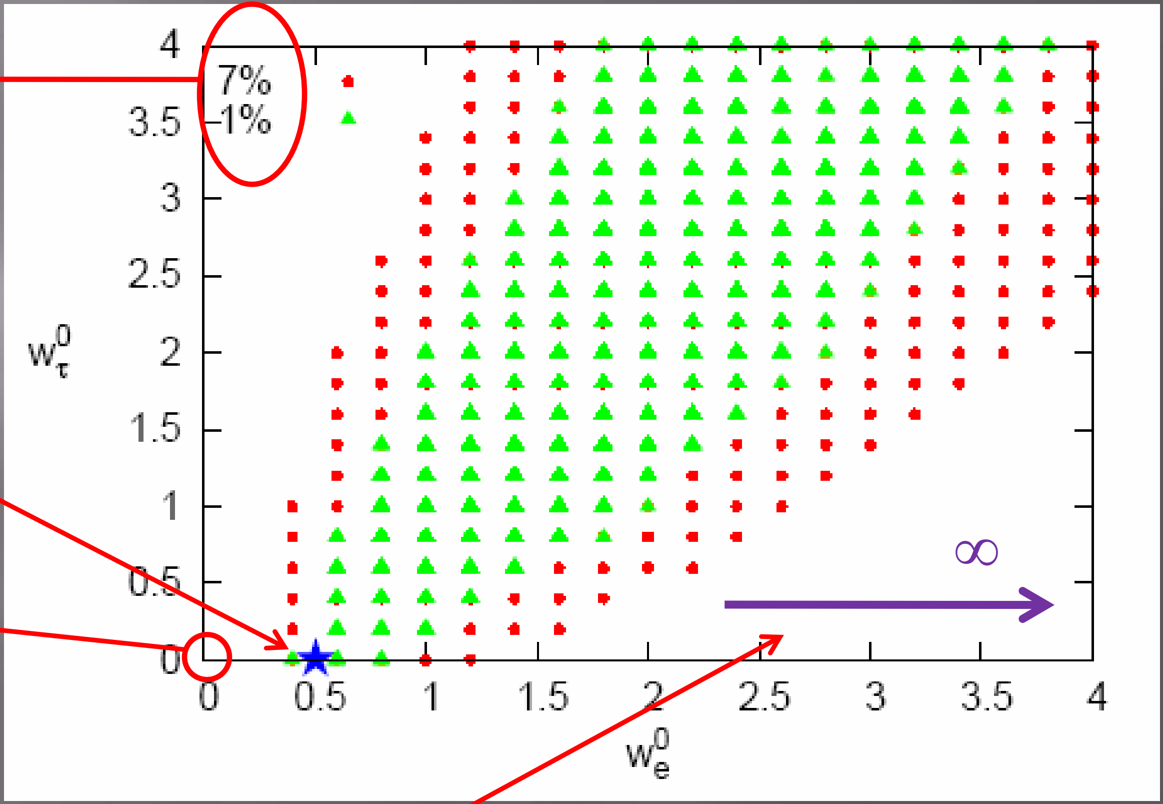
Flavor Composition at the Source

$$w_e^0 : w_\mu^0 : w_\tau^0 = w_e^0 : 1 : w_\tau^0$$

Uncertainty in the measurement of ***R***

Pion Source

Stopped-Muon Source



Neutron Source

$\sin^2\theta_{13} \in (0, 0.003)$

$\delta \in (0, 2\pi)$

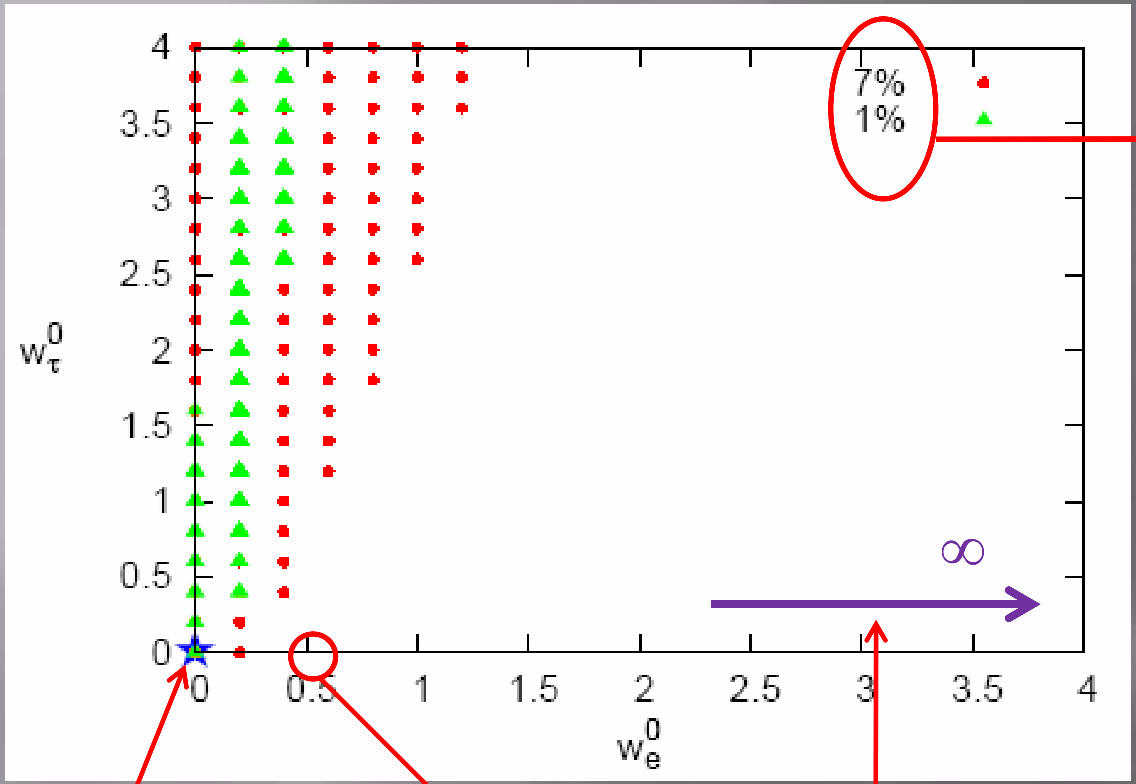
$\alpha \in (1.8, 2.2)$

$\mathcal{N}_{\bar{\nu}_e}/\mathcal{N}_{\nu_e} \in (0, 1)$

- By 7 % precision in the measurement of ***R*** , Pion source can be completely distinguished from the Stopped-muon and Neutron sources

Flavor Composition at the Source

$$w_e^0 : w_\mu^0 : w_\tau^0 = w_e^0 : 1 : w_\tau^0$$



Uncertainty in the measurement of ***R***

$\sin^2\theta_{13} \in (0, 0.003)$

$\delta \in (0, 2\pi)$

$\alpha \in (1.8, 2.2)$

$\mathcal{N}_{\bar{\nu}_e}/\mathcal{N}_{\nu_e} \in (0, 1)$

Stopped-Muon Source

Pion Source

Neutron Source

♠ By 7 % precision in the measurement of ***R*** , Stopped-Muon source can be completely distinguished from the Pion and Neutron sources

Conclusion

- For $\delta = 0$, dependence of ***R*** on $\sin^2\theta_{13}$ is very mild (~ 2 %), but for $\delta = \pi$, ***R*** changes by about 10 % by varying $\sin^2\theta_{13}$ from zero to the present upper bound
- 10 % uncertainty in the spectral index α is the main source of error in the extraction of $\sin^2\theta_{13}$ from the measurement of ***R***. By reducing this uncertainty to 5 % , it is possible to derive $\sin^2\theta_{13}$ from the measurement of ***R***
- Even with 1 % precision in the measurement of ***R*** , CP-violation cannot be established (*i. e.*; even for maximal CP violation $\delta = \pi/2$, $\delta = 0$ cannot be ruled out)
- The initial flavor ratio of neutrinos can be determined by the measurement of ***R*** . By 7 % precision, the pion (1:2:0), stopped-muon (0:1:0) and neutron (1:0:0) sources can be completely distinguished.

Back up

Sensitivity to s_{13}^2

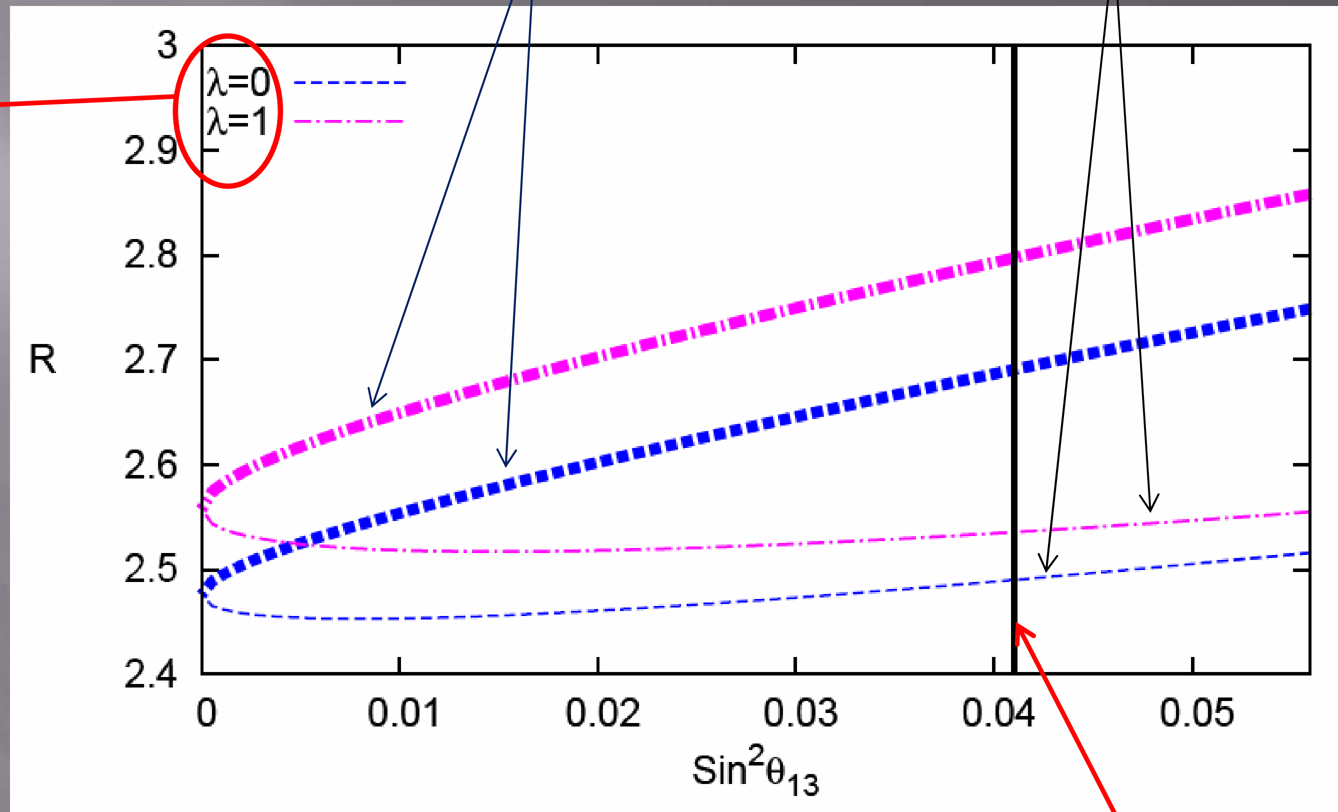
$\lambda \equiv N_{\bar{\nu}_e}/N_{\nu_e}$

$w_e^0 : w_\mu^0 : w_\tau^0 = 1 : 2 : 0$

$\alpha = 2$

θ_{12} and θ_{23}

Best fit values



The present bound on s_{13}^2 at 3σ

- Variation of λ in the interval [0,1] causes a 5 % change in ***R***
- For $s_{13}^2 > 0.005$, lack of knowledge about the content of the incoming beam will not cause a problem to distinguish $\delta = 0$ from $\delta = \pi$

- By varying $\lambda \equiv \mathcal{N}_{\bar{\nu}_e}/\mathcal{N}_{\nu_e}$ between 0 and 1, ***R*** changes by ~ 5 %, which is comparable to the effect of $\sin^2\theta_{13}$. No way to measure this ratio; we should rely on model to predict the value of this ratio.

Probability density of the emission of muon

At the rest frame of tau lepton

$$f(E_\tau, E_\mu) \equiv \frac{1}{\Gamma}\frac{d\Gamma(\tau(E_\tau) \to \mu(E_\mu)\bar{\nu}_\mu \nu_\tau)}{dE_\mu}$$

$$\frac{1}{\Gamma}\frac{d^2\Gamma}{dE_\mu d\Omega} dE_\mu d\Omega = \frac{1}{\Gamma'}\frac{d^2\Gamma'}{dE'_\mu d\Omega'} dE'_\mu d\Omega'$$

&

$$\frac{1}{\Gamma'}\frac{d^2\Gamma'}{dE'_\mu d\Omega'} = \frac{12}{\pi m_\tau^3}\left(1 - \frac{4E'_\mu}{3m_\tau}\right) E'^2_\mu$$

$$\gamma = E_\tau / m_\tau$$

$$\beta = \sqrt{1 - 1/\gamma^2}$$

$$\frac{1}{\Gamma}\frac{d^2\Gamma}{dE_\mu d\Omega} dE_\mu d\Omega = \frac{12}{\pi m_\tau^3}\left[1 - \frac{4}{3m_\tau}\gamma(1 - \beta\cos\theta)E_\mu\right]\gamma(1 - \beta\cos\theta)E_\mu^2 dE_\mu \sin\theta d\theta d\phi,$$

$$0 \le \phi < 2\pi, \qquad 0 < E_\mu < \frac{E_\tau}{2}(1+\beta), \qquad 0 \le \theta \le \theta_{max}$$

$$\theta_{max} = \arccos\left[\max\left\{\frac{1}{\beta}\left(1 - \frac{m_\tau}{2\gamma E_\mu}\right), -1\right\}\right].$$

Determination of δ

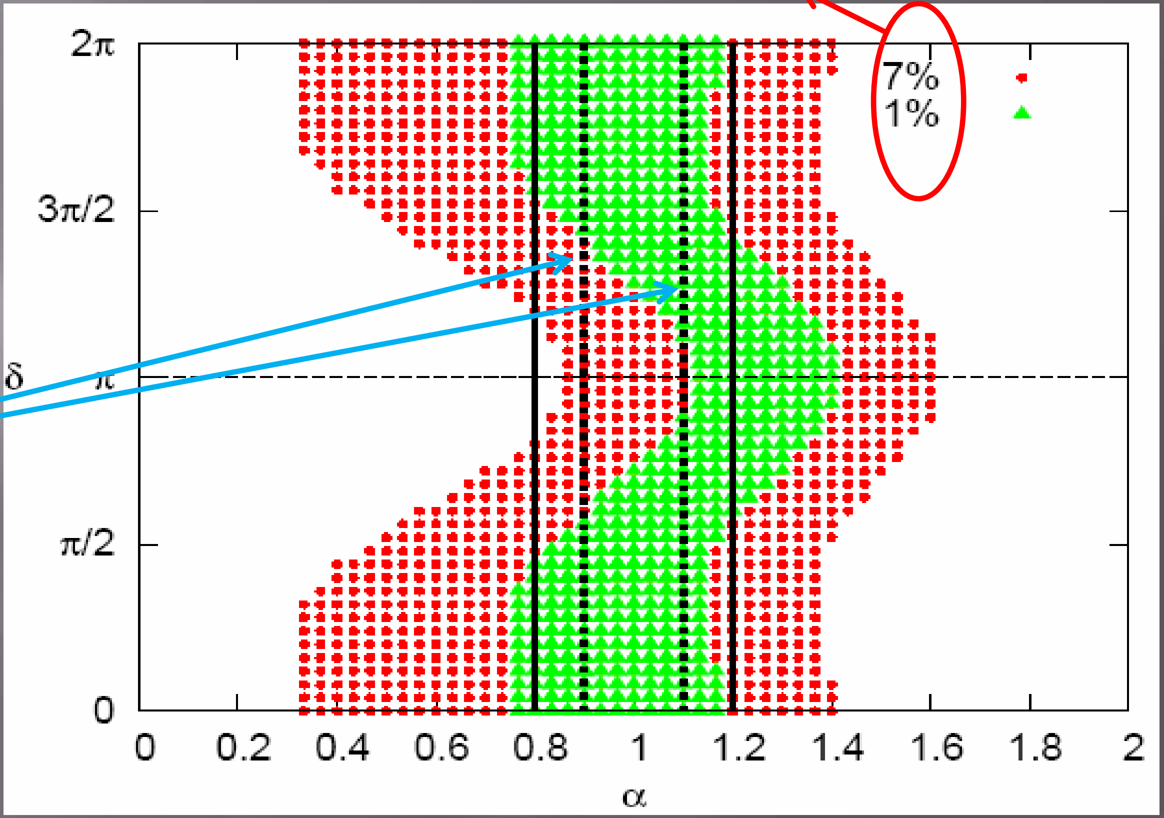
$$w_e^0 : w_\mu^0 : w_\tau^0 = 1 : 2 : 0$$

&

$$\delta = \pi/2$$

Uncertainty in the measurement of *R*

10 % precision in the measurement of α with central value α = 1



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$$\mathcal{N}_{\bar{\nu}_e}/\mathcal{N}_{\nu_e} \in (0,1)$$

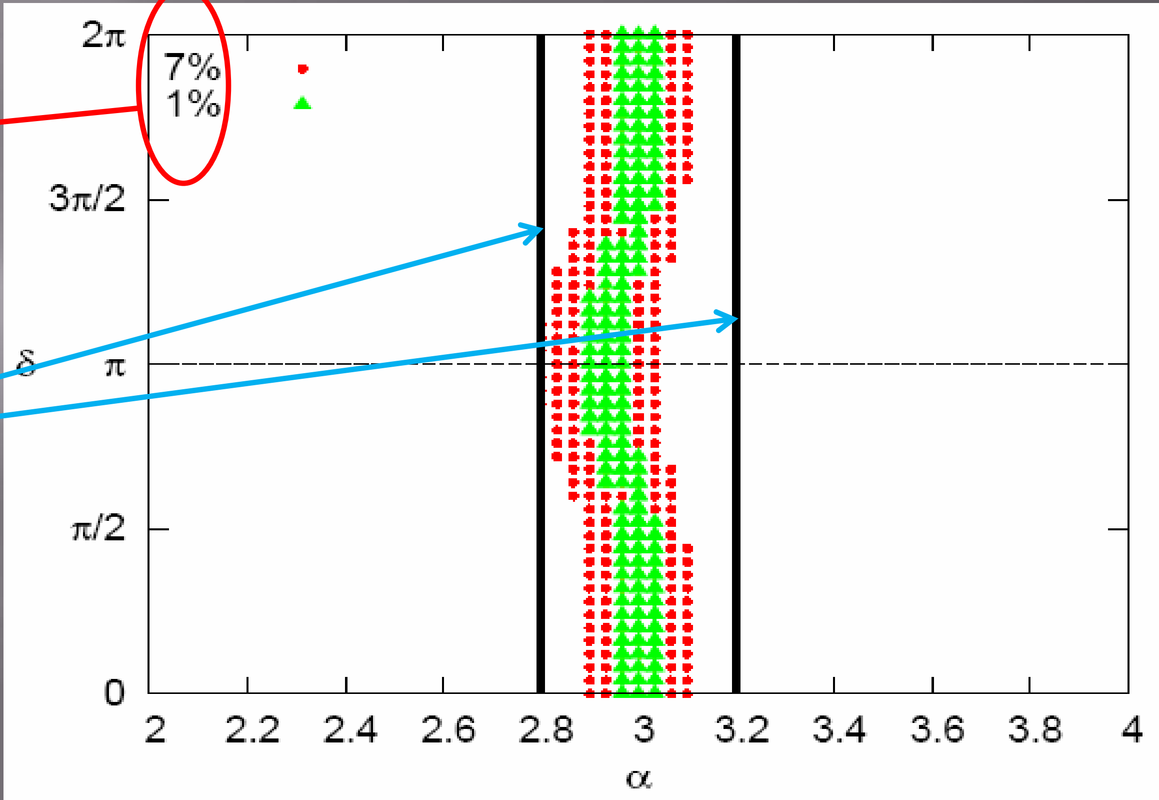
- By measuring ***R*** with 1 % precision, regions around δ = π can be excluded

Determination of δ

$\delta = \pi/2$ & $w_e^0 : w_\mu^0 : w_\tau^0 = 1 : 2 : 0$

Uncertainty in the measurement of *R*

10 % precision in the measurement of α with central value α = 3



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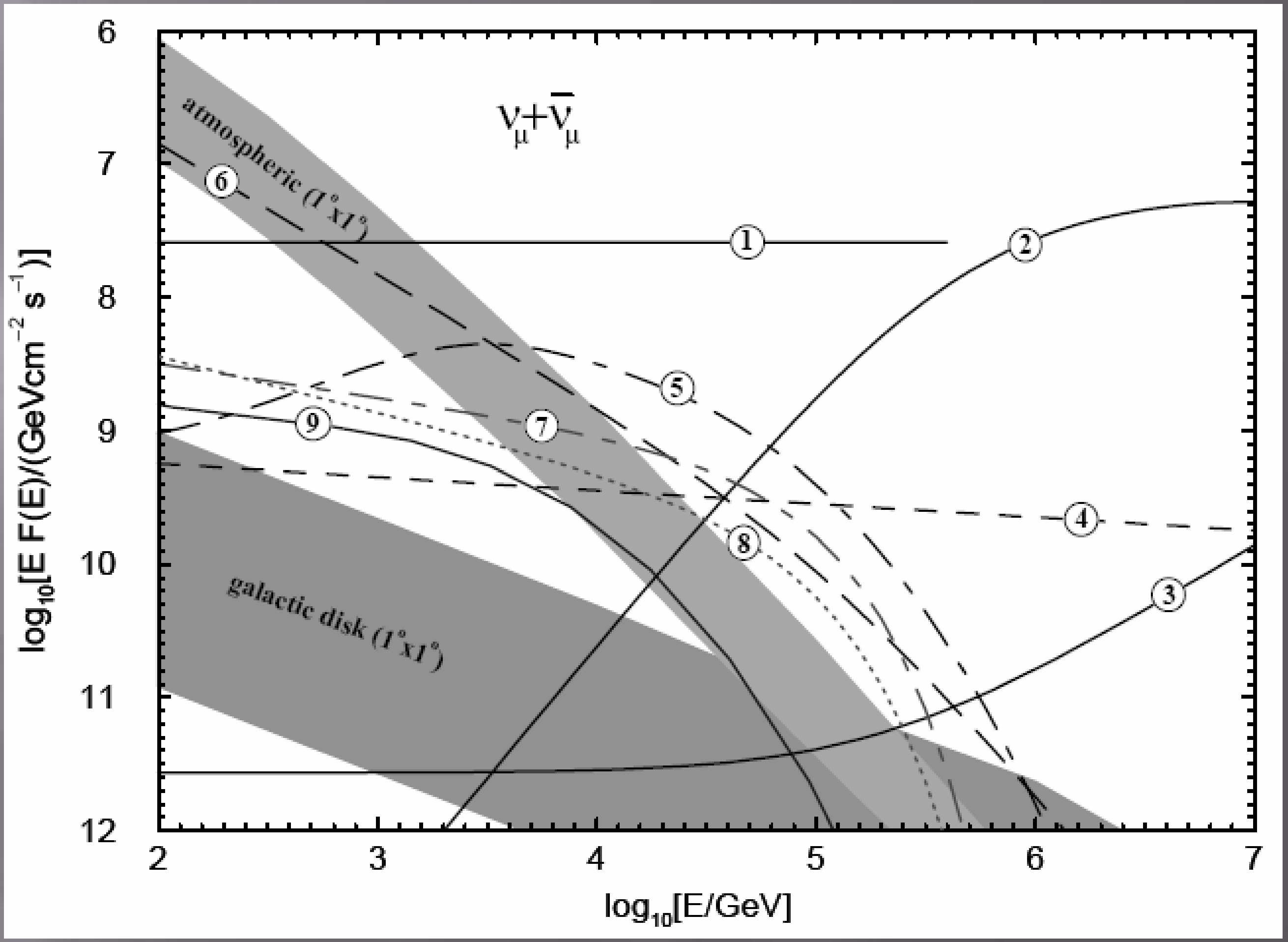
$\mathcal{N}_{\bar{\nu}_e}/\mathcal{N}_{\nu_e} \in (0, 1)$

J. G. Learned and K. Mannheim, *Ann. Rev. Nucl. Part. Sci.* 50 (2000) 679.



1. pp interaction in 3C273 Quasar
2. pγ interaction in 3C273 Quasar
3. Mannheim Model
4. Coma Cluster
5. Crab Nebula
6. Cosmic Ray induced Neutrinos from the Sun
7. SNR IC444
8. SNR γ Cygni
9. SNR Cas A